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Global evidence on the distribution of GDP growth rates[☆]

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HIGHLIGHTS

- We use global data to examine empirically the distribution of GDP growth rates.
- We use the data to test eight theoretical distributions with EDF statistics.
- The consensus finding in the literature supports the Laplace distribution.
- GDP growth rates are better fit by the Cauchy distribution, particularly for samples with fat tails.

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ABSTRACT

We study the size distribution of changes in the gross domestic product (GDP) of 167 countries for the period 1950–2011. A consensus has developed in the literature that the distribution of GDP growth rates can be approximated by the Laplace distribution in the central part and power-law distributions in the tails. Using a richer database than prior studies and testing for more theoretical distributions, we find that the distribution of GDP growth rates can be fitted using the heavy-tailed Cauchy distribution for almost all countries. Significantly, this same finding recently has been demonstrated for (1) the distribution of firm growth rates and (2) the distribution of firm economic profit rates. Together, these three findings suggest the possibility that there exist universal mechanisms that give rise to general laws governing the growth dynamics of firms and economies.

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1. Introduction

Physicists and economists have studied the possibility that there exist universal mechanisms giving rise to general laws governing the growth dynamics of firms and economies [1–4]. A consensus empirical finding has emerged that the distributions of (1) gross domestic product (GDP), (2) firm growth rates, and (3) firm economic profit rates can be approximated by the Laplace distribution in the central part and power-law distributions in the tails [5–9]. Although these distributions are reasonably well approximated by the Laplace distribution, a consistent empirical finding is that the tails are fatter than those of the Laplace distribution [3,10,11]. These empirical findings led researchers to the use of power-law distributions in the tails [3,12,13].

Recent research has shown that the distribution of firm growth rates and the distribution of firm economic profit rates are substantially better approximated by the heavy-tailed Cauchy distribution [14,15]. As a result, ad hoc adjustments to fatten the tails of the Laplace distribution with power-law tails are not required. Since the granular origins of aggregate

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fluctuations in GDP depend in substantial part on the growth dynamics of firms [4],[1] this finding points to the importance of a more detailed investigation of the size distribution of GDP growth rates [1–4].

We, therefore, explore the distribution of GDP growth rates using a richer database than used in prior studies and test more theoretical distributions. Our global database contains 167 countries over the period 1950–2011. We present empirical distribution function (EDF) tests for the growth rate of GDP in each country for eight distributions: Cauchy, exponential, gamma, Laplace, logistic, log-normal, normal, and Weibull. We find, in common with the consensus finding, that the Laplace distribution performs reasonably well in approximating the distribution of GDP growth rates. However, we show that for almost all countries, the distribution of GDP growth rates can be fitted using the heavy-tailed Cauchy distribution, a particular case of the Lévy-stable family of densities. The Lévy α-stable family ($\alpha \in (0, 2)$) includes the Gaussian, or normal, ($\alpha = 2$) and Cauchy ($\alpha = 1$) distributions as special cases. All non-Gaussian Lévy α-stable distributions (i.e., $\alpha < 2$) present heavy tails [17].

2. Data and calculation of GDP growth rates

We obtain data on GDP from the Penn World Table [18], which is a standard data source for comparing living standards across countries and explaining cross-country growth differences. The dataset contains real (inflation adjusted to 2005 dollars) GDP levels across countries over time. The GDP levels are made comparable across countries using purchasing power parities (PPP), which allows for a comparable common currency. Our dataset contains the GDP of 167 countries for the period 1950–2011. We study the logarithm of the annual growth rate in the real GDP of country i in year t:

$$g_t^i = \log(GDP_t^i) - \log(GDP_{t-1}^i). \tag{1}$$

Summary statistics on the GDP growth rates by country are presented in the Appendix, Table A.1.

3. EDF tests of theoretical distributions

To find the theoretical distribution that best approximates the empirical distributions of GDP growth rates, we perform goodness-of-fit tests using EDF test statistics. Let F denote the empirical CDF and G denote the hypothesized distribution (e.g., the normal distribution). Let (X, δ) be any metric space of CDFs. Then, the distance $\delta(G, F)$ is an EDF test statistic. Commonly used EDF test statistics include the Anderson–Darling and Cramér–von Mises test statistics [19]. The Anderson–Darling test statistic is based on the parametric family $V_n = n\int_{-\infty}^{\infty} |G(x) - F(x)|^2 \psi(x) dG(x)$, where n is the number of observations. Setting $\psi(x) = \{F(x)(1 - F(x))\}^{-1}$ yields the Anderson–Darling test statistic. The Cramér–von Mises test statistic is derived by setting $\psi(x) = 1$ in the family $V_n = n\int_{-\infty}^{\infty} |G(x) - F(x)|^2 \psi(x) dG(x)$.

The parameters of the distributions are estimated following [20,21].[2] We first test the null hypothesis that the GDP growth rates are normally distributed. As shown in Table 1, the null hypothesis is not rejected based on the Anderson–Darling test statistic in only 32.9% of the 167 country samples. Next, we test the null hypothesis that the GDP growth rates are distributed according to the Cauchy, exponential, gamma, Laplace, logistic, log-normal, normal, and Weibull distributions. At the 5% significance level, the null hypothesis that the GDP growth rates are distributed according to the Cauchy distribution is not rejected in 166 (99.4%) of the 167 country samples, while the Laplace and logistic distributions are not rejected in 75.4% and 53.3% of the samples. Using the Cramér–von Mises test statistic, the Cauchy distribution is not rejected in 99.4% of the samples, while the Laplace and logistic distributions are not rejected in 77.2% and 59.9% of the all samples. We conclude that the Cauchy distribution best fits the empirical distributions of GDP growth rates.

Prior research has shown that the Laplace distribution fits the tails of GDP growth rate distributions somewhat inaccurately distribution (see [3]). Similarly, for firm growth rates, Bottazzi et al. [10] conclude: "Whilst the Laplace distribution of [firm] growth rates was repeatedly found in previous studies and appeared to be emerging as something of a 'stylized fact,' we observe here that the growth rates of French firms are even fatter tailed than expected, a property which holds with disaggregation". Finally, for firm economic profit rates, Dosi and Nelson [11] concludes "that the rates of change in profit margins display distributions which are again fat-tailed (at least exponential, or even fatter-tailed)". These empirical findings led researchers to the use of power-law distributions in the tails [3,12,13]. As demonstrated here, such ad hoc adjustments are not necessary with the Cauchy distribution. The data reject the null hypothesis of a Cauchy distribution in only a small number of countries. Using the Anderson–Darling test, the Cauchy distribution is rejected for Swaziland. Similarly, using the Cramér–von Mises test, the Cauchy distribution is rejected for Swaziland.

The Cauchy distribution is a stable, "fat-tailed" distribution with an infinite variance. This variance condition has an interesting economic implication: both very low and very high GDP growth rates will occur much more frequently than predicted by the normal (Gaussian) distribution. For example, in the domain of observations three or more standard deviations away from the mean of the standard normal distribution, the Cauchy distribution with location 0 and scale 1 has

[1] See also [16], who develop an agent-based model that yields a tent-shaped aggregate distribution of firm growth rates.

[2] In [20,21], parameters for exponential, gamma, Laplace, logistic, and Weibull distributions are estimated using maximum likelihood. For maximum likelihood estimators with no analytical expressions, we use the Nelder–Mead method for direct optimization of the log-likelihood function.

Table 1
GDP growth rates: Percentage of country samples not rejected.

Distribution	Anderson–Darling test	Cramér–von Mises test
Cauchy	99.4	99.4
Exponential	0.0	0.0
Gamma	5.4	8.4
Laplace	75.4	77.2
Logistic	53.3	59.9
Log-normal	0.6	0.6
Normal	32.9	33.5
Weibull	21.0	24.6
Number of samples	167	167
Time period	1950–2011	1950–2011

Note: Statistical significance is determined at the 5% level.

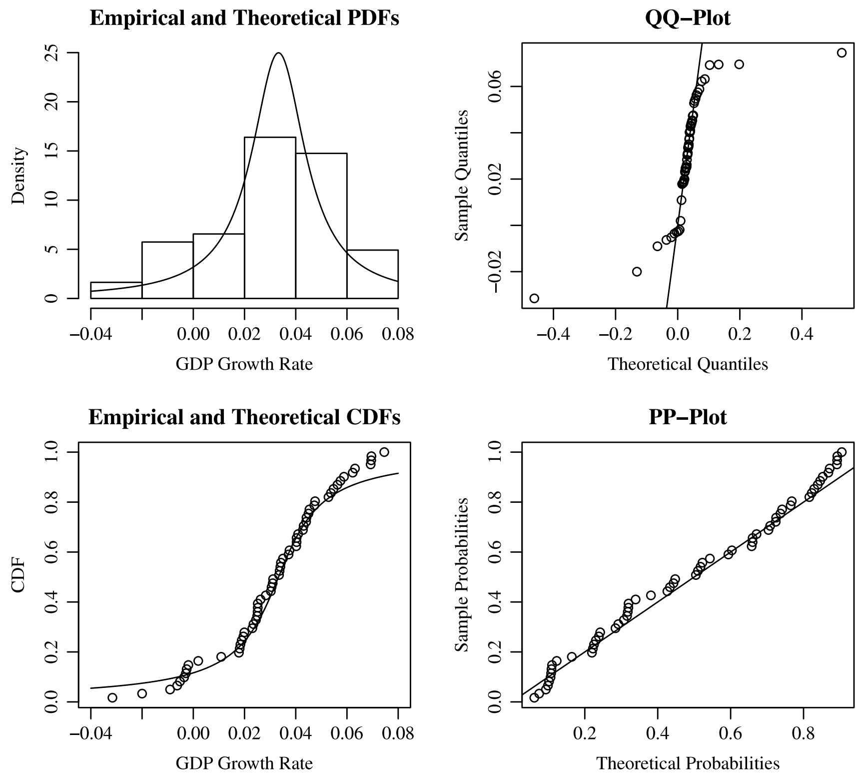

Fig. 1. P–P and Q–Q plots and the empirical and theoretical Cauchy densities and distributions for US GDP growth rates.

in excess of 75 times the percentage of observations than the corresponding percentage for the standard normal distribution. At four or more standard deviations away from the mean, the Cauchy distribution has more than 2000 times the percentage of observations.

Probability–probability (P–P) plots and quantile–quantile (Q–Q) plots of the empirical and theoretical distributions provide additional goodness-of-fit measures and show further that the Cauchy distribution describes the distribution of GDP growth rates. If $F(\cdot)$ and $G(\cdot)$ represent two distribution functions with the same support, then the P–P plot in functional form equals $F(G^{-1}(p))$, where $0 < p < 1$. The Q–Q plot equals $F^{-1}(G(x))$ for all real x (see [22]). For example, Fig. 1 presents the P–P and Q–Q plots and the empirical and theoretical Cauchy densities and distributions for the US GDP growth rate. The Cauchy distribution is parameterized by the maximum-likelihood location and scale parameters. Both diagrams show clustering along the relevant 45-degree line, providing empirical evidence that the data are distributed according to the estimated Cauchy distribution.

In addition to finding the theoretical distribution that best approximates the empirical distributions of GDP growth rates, we also perform goodness-of-fit tests using EDF test statistics on the size distribution of GDP per capita growth rates. Using the same database, the GDP per capita is calculated by dividing real GDP in a given country and year by the corresponding population. We then study the logarithm of the annual growth rate in the real GDP per capita for the given country and year. Testing the null hypothesis that the GDP per capita growth rates are normally distributed, Table 2 shows the null

Table 2
GDP per capita growth rates: Percentage of country samples not rejected.

Distribution	Anderson–Darling test	Cramér–von Mises test
Cauchy	99.4	100.0
Exponential	0.0	0.0
Gamma	4.8	6.6
Laplace	76.6	82.6
Logistic	55.7	62.3
Log-normal	1.2	1.2
Normal	33.5	35.3
Weibull	22.8	24.6
Number of samples	167	167
Time period	1950–2011	1950–2011

Note: Statistical significance is determined at the 5% level.

Table 3
Vuong likelihood ratio test: Country samples for which the Anderson–Darling test does not reject either the Laplace or the Cauchy distribution for GDP growth rates.

Growth rate variable	Total	Cauchy is preferred	Laplace is preferred	No statistical difference
GDP	126 100%	0 0.0%	81 64.3%	45 35.7%
GDP per capita	128 100%	0 0.0%	87 68.0%	41 32.0%

Note: 1. Number of countries and percentages as of total are reported.
2. Statistical significance is determined at the 5% level.

hypotheses is not rejected based on the Anderson–Darling test statistic in only 33.5% of the 167 country samples. Next, we test the null hypothesis that the GDP per capita growth rates are distributed according to the Cauchy, exponential, gamma, Laplace, logistic, log-normal, normal, and Weibull distributions [23]. At the 5% significance level, the null hypothesis that the GDP per capita growth rates are distributed according to the Cauchy distribution is not rejected in 99.4% of the 167 country samples, while the Laplace and logistic distributions are not rejected in 76.6% and 55.7% of the samples. Using the Cramér–von Mises test statistic, the Cauchy distribution is not rejected in 100.0% of the samples, while the Laplace and logistic distributions are not rejected in 82.6% and 62.3% of the all samples. We conclude that the Cauchy distribution best fits the empirical distributions of GDP per capita growth rates.

4. Comparing the Cauchy and Laplace distributions

Tables 1 and 2 show that the Anderson–Darling and Cramér–von Mises tests almost never reject the Cauchy distribution, but reject the Laplace distribution for 17%–25% of the country samples. In this sense, the Cauchy distribution is a safer modeling choice than the Laplace distribution in the absence of *ex ante* knowledge of the underlying distribution of the population. However, the possibility exists that the Laplace distribution could offer better fits for specific types of country samples. We investigate that possibility here.

For those country samples for which an EDF test rejects the Laplace distribution but does not reject the Cauchy distribution, the distributions of the GDP growth rate and GDP per capita growth rate are best approximated by the Cauchy distribution. However, for those country samples for which an EDF test does not reject either the Laplace or the Cauchy distribution, the test does not reveal which distribution fits a specific country sample better.[3] In this case, since the Laplace and Cauchy distributions are not nested, we use the test specified by Vuong [24] to determine whether the Laplace distribution or the Cauchy distribution fits a country sample better. As shown in Table 3, at the 5% significance level, among the 126 countries for which the Anderson–Darling test does not reject either the Cauchy or the Laplace distribution, the Vuong test shows that the Laplace distribution provides a better fit than the Cauchy distribution for approximately two thirds of these 126 countries. For the remaining one third of these countries, one cannot discriminate between the two competing distributions given the data.[4]

In order to further investigate these results, we examine two subsets of country samples. The first subset consists of those 40 country samples for which the Anderson–Darling test rejects the Laplace distribution for GDP growth rates but does not reject the Cauchy distribution (the Cauchy-preferred group). The second subset consists of those 81 country samples for

[3] We thank an anonymous referee for suggesting that we apply a direct test to compare the Laplace distribution against the Cauchy distribution in this case.

[4] For the countries for which the Cramér–von Mises test does not reject either the Cauchy or the Laplace distribution, the Vuong test generates similar results.

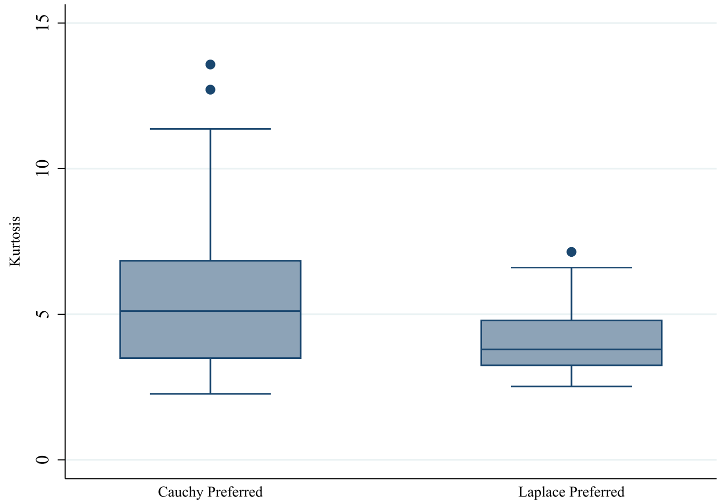

Fig. 2. Comparison of kurtosis between two groups of countries—GDP growth rate.

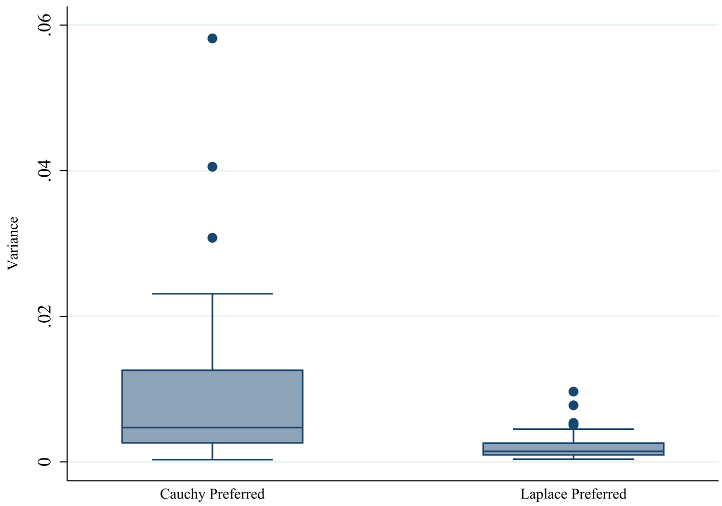

Fig. 3. Comparison of variance between two groups of countries—GDP growth rate.

which the Anderson–Darling test rejects neither the Laplace distribution nor the Cauchy distribution for GDP growth rates, but the Laplace distribution is preferred according to the Vuong test (the Laplace-preferred group).[5]

With respect to these two groups, we compute the kurtosis and variance for the GDP growth rate of a country and compare these moments across the two groups of countries (see Figs. 2 and 3).[6] The figures show that country samples in the Cauchy-preferred group tend to have fatter tails and larger variances than those in the Laplace-preferred group. The results are similar if we define the groups based on results of the Cramér–von Mises test instead of the Anderson–Darling test.

With respect to GDP per capita growth rates, Figs. 4 and 5 compare the kurtosis and variance between the two groups. The figures show, again, that country samples in the Cauchy-preferred group tend to have fatter tails and larger variances than those in the Laplace-preferred group. The results are similar if we define the groups based on results of the Cramér–von Mises test instead of the Anderson–Darling test.

5 There is no country sample for which the Anderson–Darling test rejects the Cauchy distribution but does not reject the Laplace distribution.

6 In each figure, there are two vertical box plots. For each box plot, the bar in the middle represents the median, and the upper and lower hinges are the 75th and 25th percentiles, respectively. The upper and lower adjacent values are as defined by Tukey [25]: Let x represent a variable for which adjacent values are being calculated. Define $x_{(i)}$ as the ith ordered value of x, and define $x_{[25]}$ and $x_{[75]}$ as the 25th and 75th percentiles. Define U as $x_{[75]} + \frac{3}{2}(x_{[75]} - x_{[25]})$. The upper adjacent value is defined as x_i, such that $x_{(i)} \leq U$ and $x_{(i+1)} > U$. Define L as $x_{[25]} - \frac{3}{2}(x_{[75]} - x_{[25]})$. The lower adjacent value is defined as x_i, such that $x_{(i)} \geq L$ and $x_{(i-1)} < L$. The dots are outside values. See STATA Manual at http://www.stata.com/manuals14/g-2graphbox.pdf.

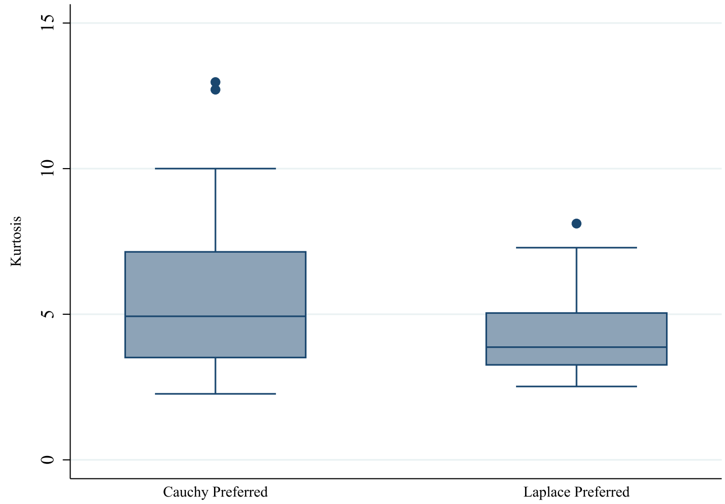

Fig. 4. Comparison of kurtosis between two groups of countries—GDP per capita growth rate.

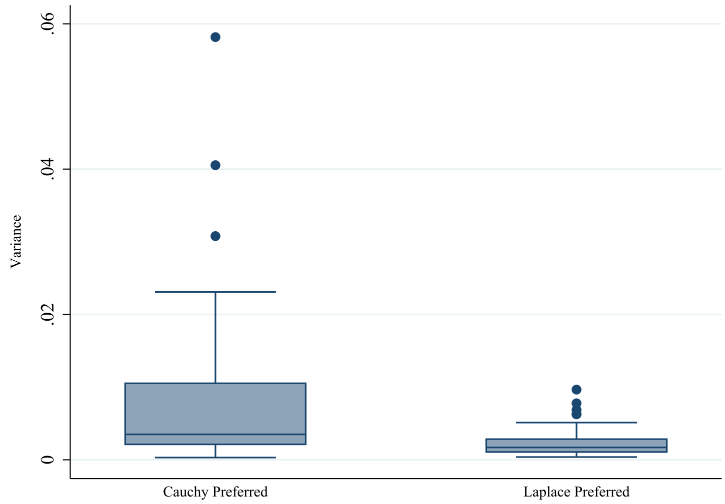

Fig. 5. Comparison of variance between two groups of countries—GDP per capita growth rate.

5. Conclusions

EDF testing rejects the hypothesis that the distributions of GDP growth rates and GDP per capita growth rates is Gaussian. We find that the Cauchy distribution fits those distributions for almost all countries. When EDF tests do not reject either the Laplace distribution or the Cauchy distribution, the Laplace distribution is more likely to fit those distributions better than the Cauchy distribution. However, modeling the distribution of GDP growth rates or GDP per capita growth rates based on the Laplace distribution is risky in the sense that the Laplace distribution is rejected by EDF tests in 17%–25% of the country samples, while the Cauchy distribution is almost never rejected. In particular, the Cauchy distribution is more likely to fit better than the Laplace distribution when the data exhibit fat tails.

As Dosi et al. [26] conclude, these results have an important economic implication: "this revealed structure in the stochastic process describing industrial evolution bears clear signs common to all complex system dynamics including, the fat-tailed distributions in the rates of changes of all the variable of interest. That, in turn, is likely to witness for the existence of some underlying correlation mechanism, which makes the system self-organized in its growth process. In these respects, all the evidence on industrial change corroborates the exciting conjecture that evolutionary phenomena tend to undergo non-Gaussian lives influenced by persistent positive or negative interactions among agents within and across the relevant populations". Our results invite research on dynamic models of competition in which firms' growth rates, economic profit rates, and resulting changes in GDP follow a Cauchy distribution.

Appendix

See Table A.1.

Table A.1
Summary statistics by country—annual GDP percentage growth rates.

Country	Number of observations	Min. Year	Max. Year	Median	Mean	Stand. Dev.	Min.	Max.
Albania	41	1971	2011	4.0	3.0	7.6	−32.9	12.7
Angola	41	1971	2011	3.1	3.7	8.3	−27.4	20.9
Antigua and Barbuda	41	1971	2011	5.0	3.5	5.6	−12.6	12.7
Argentina	61	1951	2011	3.7	2.9	5.2	−11.5	10.3
Armenia	21	1991	2011	5.7	2.1	15.2	−54.1	14.0
Australia	61	1951	2011	3.8	3.5	2.3	−4.2	10.3
Austria	61	1951	2011	3.2	3.3	2.4	−3.9	10.8
Azerbaijan	21	1991	2011	8.8	4.1	14.9	−26.3	29.6
Bahamas	41	1971	2011	2.7	2.2	7.1	−18.3	23.2
Bahrain	41	1971	2011	4.5	4.3	6.6	−17.5	26.6
Bangladesh	52	1960	2011	4.7	3.5	4.1	−15.1	9.2
Barbados	51	1961	2011	2.6	2.3	4.2	−7.3	13.9
Belarus	21	1991	2011	5.1	3.1	7.4	−12.4	10.8
Belgium	61	1951	2011	2.9	2.8	1.9	−2.8	6.8
Belize	41	1971	2011	4.1	4.4	3.6	−2.2	12.2
Benin	52	1960	2011	4.2	3.8	3.3	−7.1	10.2
Bermuda	41	1971	2011	2.4	2.0	2.9	−4.9	8.6
Bhutan	41	1971	2011	6.0	6.7	4.8	−3.4	27.3
Bolivia	61	1951	2011	3.7	2.5	4.6	−17.4	9.5
Bosnia and Herzegovina	21	1991	2011	5.4	7.5	12.0	−9.8	43.3
Botswana	51	1961	2011	7.7	8.4	6.2	−4.9	28.7
Brazil	61	1951	2011	5.0	4.9	3.8	−4.4	13.2
Brunei	41	1971	2011	2.6	2.4	6.9	−22.1	20.3
Bulgaria	41	1971	2011	4.5	2.8	5.2	−9.9	10.4
Burkina Faso	52	1960	2011	4.1	3.5	5.1	−14.0	20.2
Burundi	51	1961	2011	3.4	2.8	6.0	−15.4	20.8
Cambodia	41	1971	2011	4.8	3.1	7.9	−21.0	19.5
Cameroon	51	1961	2011	3.6	3.0	5.2	−11.7	19.9
Canada	61	1951	2011	3.4	3.5	2.4	−2.9	8.9
Cape Verde	51	1961	2011	5.5	4.9	6.0	−18.5	19.3
Central African Republic	51	1961	2011	1.8	1.3	3.7	−7.8	7.8
Chad	51	1961	2011	1.9	3.5	8.3	−24.1	29.0
Chile	60	1952	2011	4.8	4.0	5.0	−14.6	11.6
China	59	1953	2011	8.7	7.4	5.6	−20.9	14.1
Colombia	61	1951	2011	4.2	4.2	2.3	−4.3	8.1
Comoros	51	1961	2011	2.9	3.4	3.7	−5.4	12.9
Congo, Dem. Rep.	61	1951	2011	2.1	1.4	6.0	−14.5	13.6
Congo, Republic of	51	1961	2011	4.2	4.5	6.2	−7.7	21.2
Costa Rica	61	1951	2011	5.4	5.0	3.7	−7.6	16.1
Cote d'Ivoire	51	1961	2011	3.0	3.6	5.5	−11.6	17.3
Croatia	21	1991	2011	3.7	0.4	7.6	−23.7	6.6
Cyprus	61	1951	2011	5.0	4.6	7.0	−21.1	19.3
Czech Republic	21	1991	2011	2.9	1.9	4.3	−12.3	6.8
Denmark	61	1951	2011	2.4	2.5	2.6	−6.0	8.1
Djibouti	41	1971	2011	3.0	2.2	4.6	−12.6	12.0
Dominica	41	1971	2011	3.2	3.5	5.3	−20.5	13.4
Dominican Republic	60	1952	2011	5.4	5.3	4.7	−11.5	17.0
Ecuador	60	1952	2011	4.2	4.5	4.0	−6.2	22.6
Egypt	61	1951	2011	5.0	5.4	3.7	−2.7	17.3
El Salvador	61	1951	2011	3.7	3.2	3.5	−9.1	10.1
Equatorial Guinea	51	1961	2011	4.9	8.3	13.0	−5.1	66.9
Estonia	21	1991	2011	4.4	1.7	8.9	−23.8	10.5
Ethiopia	61	1951	2011	3.5	4.0	6.6	−13.6	28.5
Fiji	51	1961	2011	2.6	3.2	4.5	−6.7	12.8
Finland	61	1951	2011	3.2	3.3	3.3	−8.9	9.2
France	61	1951	2011	3.2	3.2	2.1	−3.2	7.8
Gabon	51	1961	2011	5.1	4.3	9.8	−27.5	33.3
Gambia, The	51	1961	2011	3.3	3.3	3.7	−6.0	16.6
Georgia	21	1991	2011	4.7	−1.4	17.5	−58.9	11.6
Germany	61	1951	2011	3.0	3.4	2.9	−5.3	11.4
Ghana	56	1956	2011	4.6	3.6	5.1	−13.3	19.8
Greece	60	1952	2011	3.5	3.5	4.2	−7.2	12.4
Grenada	41	1971	2011	4.4	3.9	4.2	−7.0	11.4
Guatemala	61	1951	2011	3.9	3.8	2.4	−3.6	9.1
Guinea	52	1960	2011	2.9	2.7	3.6	−9.3	13.9

Table A.1 (*continued*)

Country	Number of observations	Min. Year	Max. Year	Median	Mean	Stand. Dev.	Min.	Max.
Guinea-Bissau	51	1961	2011	3.2	1.2	8.4	−31.7	14.6
Honduras	61	1951	2011	4.0	3.5	4.3	−13.5	17.3
Hong Kong	51	1961	2011	5.9	6.2	4.5	−6.2	15.0
Hungary	41	1971	2011	2.8	2.0	3.7	−12.7	6.7
Iceland	61	1951	2011	4.0	3.8	4.3	−6.8	13.9
India	61	1951	2011	5.3	4.9	3.1	−5.2	10.6
Indonesia	51	1961	2011	6.0	5.2	3.9	−14.1	10.7
Iran	56	1956	2011	5.9	4.4	9.9	−49.8	18.2
Iraq	41	1971	2011	5.6	4.1	24.1	−108.2	43.3
Ireland	61	1951	2011	3.8	3.7	3.4	−5.6	10.9
Israel	61	1951	2011	5.3	6.3	6.6	−0.9	43.5
Italy	61	1951	2011	3.1	3.2	2.9	−5.7	11.8
Jamaica	58	1954	2011	1.5	2.4	4.2	−6.7	12.8
Japan	61	1951	2011	4.3	5.0	5.1	−5.7	30.9
Jordan	57	1955	2011	5.1	5.3	7.9	−18.0	33.8
Kazakhstan	21	1991	2011	3.2	2.4	8.0	−13.4	12.7
Kenya	61	1951	2011	4.2	3.7	3.8	−5.9	16.1
Korea, Republic of	58	1954	2011	7.0	6.7	3.8	−5.9	13.8
Kuwait	41	1971	2011	4.1	1.2	15.6	−52.8	41.0
Kyrgyzstan	21	1991	2011	3.1	0.2	8.9	−22.4	9.5
Laos	41	1971	2011	6.5	5.6	3.4	−2.0	14.3
Latvia	21	1991	2011	5.3	0.2	12.1	−38.7	10.6
Lebanon	41	1971	2011	4.5	1.7	24.1	−84.4	60.6
Lesotho	51	1961	2011	4.1	4.6	6.3	−11.5	23.2
Liberia	47	1965	2011	3.6	0.8	20.1	−71.4	72.4
Lithuania	21	1991	2011	5.1	0.8	9.8	−23.9	9.8
Luxembourg	61	1951	2011	3.5	3.4	3.4	−6.8	10.2
Macao	41	1971	2011	7.6	7.4	5.9	−4.7	23.9
Macedonia	21	1991	2011	1.8	1.2	3.7	−8.5	6.0
Madagascar	51	1961	2011	2.1	1.8	4.5	−13.5	11.2
Malawi	57	1955	2011	4.4	4.7	7.2	−12.3	24.5
Malaysia	56	1956	2011	6.6	6.2	4.0	−7.6	20.6
Maldives	41	1971	2011	8.4	7.5	6.7	−9.1	23.9
Mali	51	1961	2011	4.2	3.5	5.3	−17.0	14.9
Malta	57	1955	2011	4.6	4.8	4.4	−4.3	17.9
Mauritania	51	1961	2011	2.8	4.2	7.8	−7.9	43.2
Mauritius	61	1951	2011	4.2	3.8	4.9	−14.5	20.0
Mexico	61	1951	2011	4.2	4.3	3.8	−6.4	12.8
Moldova	21	1991	2011	2.1	−2.3	12.9	−37.0	7.5
Mongolia	41	1971	2011	5.0	4.5	4.8	−10.0	15.9
Montenegro	21	1991	2011	2.5	0.4	14.7	−45.5	24.4
Morocco	61	1951	2011	4.2	4.2	4.9	−6.9	18.4
Mozambique	51	1961	2011	4.8	4.2	5.1	−14.1	13.8
Namibia	51	1961	2011	3.9	3.9	3.4	−2.5	13.0
Nepal	51	1961	2011	3.9	3.6	2.7	−3.0	9.2
Netherlands	61	1951	2011	3.0	3.3	2.5	−3.7	9.4
New Zealand	61	1951	2011	2.7	2.7	3.4	−5.5	14.7
Niger	51	1961	2011	1.6	2.0	6.1	−18.7	12.6
Nigeria	61	1951	2011	4.9	3.7	7.3	−17.1	21.8
Norway	61	1951	2011	3.7	3.3	1.8	−1.7	6.7
Oman	41	1971	2011	5.4	6.8	7.2	−15.1	25.1
Pakistan	61	1951	2011	4.8	4.7	2.4	−0.9	10.7
Panama	61	1951	2011	5.3	5.5	4.2	−10.7	19.5
Paraguay	60	1952	2011	4.0	4.0	3.8	−3.9	14.0
Peru	61	1951	2011	4.7	3.8	4.8	−13.5	12.1
Philippines	61	1951	2011	4.9	4.3	3.0	−7.6	10.0
Poland	41	1971	2011	4.2	3.0	4.7	−12.3	7.6
Portugal	61	1951	2011	4.0	3.7	3.2	−4.4	12.1
Qatar	41	1971	2011	5.7	6.6	7.2	−7.6	26.2
Romania	51	1961	2011	5.5	4.1	5.9	−13.8	14.3
Russia	21	1991	2011	4.2	0.5	7.6	−15.7	9.6
Rwanda	51	1961	2011	5.3	3.4	12.2	−67.0	28.9
Sao Tome and Principe	41	1971	2011	2.3	2.8	5.4	−10.9	21.0
Saudi Arabia	41	1971	2011	4.1	4.1	6.7	−11.0	20.4
Senegal	51	1961	2011	3.4	2.7	4.1	−7.2	14.3
Serbia	21	1991	2011	3.5	−1.4	12.3	−36.3	8.9
Sierra Leone	50	1962	2011	3.4	2.3	8.2	−28.5	23.3
Singapore	51	1961	2011	8.2	7.4	4.0	−3.9	13.8

(*continued on next page*)

Table A.1 (*continued*)

Country	Number of observations	Min. Year	Max. Year	Median	Mean	Stand. Dev.	Min.	Max.
Slovak Republic	21	1991	2011	4.3	2.7	5.7	−15.7	10.0
Slovenia	21	1991	2011	3.6	2.2	4.4	−9.3	6.7
South Africa	61	1951	2011	3.6	3.3	2.3	−2.2	7.7
Spain	61	1951	2011	3.7	4.0	3.7	−3.8	15.6
Sri Lanka	61	1951	2011	5.0	4.6	3.4	−12.9	11.1
St. Kitts and Nevis	41	1971	2011	5.0	4.2	3.3	−7.0	9.9
St. Lucia	41	1971	2011	2.9	4.0	4.6	−5.8	17.2
St. Vincent and Grenadines	41	1971	2011	3.6	3.5	5.6	−11.8	23.0
Sudan	41	1971	2011	5.8	4.5	5.4	−6.5	15.4
Suriname	41	1971	2011	3.8	2.3	5.3	−9.0	13.1
Swaziland	41	1971	2011	2.9	4.7	5.4	−5.0	22.7
Sweden	61	1951	2011	2.8	2.7	2.1	−5.2	6.6
Switzerland	61	1951	2011	2.7	2.5	2.7	−7.6	7.8
Syria	51	1961	2011	5.0	5.0	8.8	−18.7	31.5
Taiwan	60	1952	2011	7.2	7.2	3.1	−1.8	12.7
Tajikistan	21	1991	2011	6.3	−0.6	13.1	−34.2	10.5
Tanzania	51	1961	2011	4.4	4.5	3.3	−4.8	18.3
Thailand	61	1951	2011	5.7	5.9	5.9	−10.2	32.2
Togo	51	1961	2011	3.6	3.0	5.6	−17.8	13.6
Trinidad and Tobago	61	1951	2011	3.9	4.3	5.3	−10.9	14.2
Tunisia	51	1961	2011	4.7	5.1	4.4	−2.0	26.5
Turkey	61	1951	2011	5.4	4.8	5.3	−10.0	21.2
Turkmenistan	21	1991	2011	5.3	3.1	9.8	−19.0	15.3
Uganda	61	1951	2011	5.4	4.0	4.7	−13.4	12.1
Ukraine	21	1991	2011	2.3	−1.7	10.3	−26.1	11.5
United Kingdom	61	1951	2011	2.7	2.3	2.1	−4.1	7.1
United States	61	1951	2011	3.3	3.1	2.3	−3.2	7.5
Uruguay	61	1951	2011	2.6	2.3	4.5	−9.9	14.4
Uzbekistan	21	1991	2011	4.3	3.5	5.2	−11.9	9.1
Venezuela	61	1951	2011	4.3	3.8	5.5	−9.3	16.8
Vietnam	41	1971	2011	6.5	5.9	3.0	−3.6	12.7
Yemen	22	1990	2011	4.7	5.6	6.0	−11.1	18.7
Zambia	56	1956	2011	2.9	2.9	5.0	−9.7	15.6
Zimbabwe	57	1955	2011	2.1	3.0	7.1	−12.7	29.2

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