Leveling the Playing Field? The Role of Public Campaign Funding in Elections

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In a series of First Amendment cases, the U.S. Supreme Court established that government may regulate campaign finance, but not if regulation imposes costs on political speech and the purpose of regulation is to "level the political playing field." The Court has applied this principle to limit the ways in which governments can provide public campaign funding to candidates in elections. A notable example is the Court's decision to strike down matching funds provisions of public funding programs (*Arizona Free Enterprise Club's Freedom Club PAC v. Bennett*, 2011). In this paper, we develop a contest-theoretic model of elections in which we analyze the effects of public campaign funding mechanisms, including a simple public option and a public option with matching funds, on program participation, political speech, and election outcomes. We show that a public option with matching funds is equivalent to a simple public option with a lump-sum transfer equal to the maximum level of funding under the matching program; that a public option does not always "level the playing field," but may make it more uneven and can decrease as well as increase the quantity of political speech by all candidates, depending on the maximum public funding level; and that a public option tends to increase speech in cases where it levels the playing field. Several of the Supreme Court's arguments in *Arizona Free Enterprise* are discussed in light of our theoretical results. (*JEL*: D72, H41, H71, H76, K19)

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1. Introduction

Half of the states in the United States provide some form of public funding to political campaigns in state elections, and sixteen of these states allocate public funds directly to candidates. As a general rule, political candidates who accept public funding are required to limit their campaign spending and restrict their private fundraising activities.[1] The goals of public campaign funding include curbing political corruption and helping less wealthy candidates remain competitive in races against well-funded opponents.

In a series of highly publicized cases, however, the U.S. Supreme Court has placed limits on the interests that government can legitimately pursue when regulating campaign finance, including restrictions on the ways in which public campaign funding can be provided. In particular, the Court has established that government may regulate campaign finance, but not if regulation imposes costs on political speech and the primary purpose of regulation is to "level the political playing field."[2] This doctrine has affected public campaign financing programs in several U.S. states. In particular, public financing programs that try to achieve a financial balance in elections by allocating state funds to participating candidates in direct response to campaign spending by non-participating candidates are unconstitutional, as they impose an unjustifiable burden on the speech of the latter.

In this paper, we examine formally the effects of different public campaign financing mechanisms on political speech and election outcomes. To do so, we develop a contest-theoretic model in which two candidates compete by engaging in costly political speech. The winning candidate is

1. The strongest form of these restrictions is imposed by *clean elections* systems, which prohibit publicly funded candidates from accepting any private donations. Seven states have adopted clean elections laws to date: Arizona, Connecticut, Maine, New Jersey, New Mexico, North Carolina, and Vermont. Source: National Conference of State Legislatures (www.ncsl.org).

2. Landmark decisions that were based on the "leveling the playing field" argument include *Davis v. FEC* (2008), which concerns individual private campaign contribution limits; *Citizens United v. FEC* (2010), which concerns the regulation of independent political expenditures; *Arizona Free Enterprise Club's Freedom Club PAC v. Bennett* (2011), which concerns the provision of public campaign funds and which we discuss in more detail in Section 5 of this paper; and *McCutcheon v. FEC* (2014), which concerns aggregate private campaign contribution limits.

determined through a *Tullock success function*, so that the likelihood of election increases in a candidate's own speech and decreases in the speech of the opponent. Candidates differ in their costs of raising private funds, reflecting differences in wealth or access to wealthy donors. We then introduce public campaign funding to this framework. Participation in a public program frees a candidate from the need to raise private funds, but limits his speech to the level feasible with public funds.

Our model encompasses a variety of public funding mechanisms. One such mechanism is a *simple public option*: Candidates who participate in the program receive a one-time, lump-sum transfer of state funds to be used in their campaigns, but they are barred from raising private funds. This is the most common form of public funding used in the U.S. states. Another mechanism is a *public option with matching funds* (also called *trigger funds*). This mechanism works as follows: Candidates who opt for public financing first receive an initial distribution of state funds to their campaigns. If campaign spending by privately funded candidates exceeds this initial public outlay, additional matching transfers are made to publicly funded candidates, up to a predetermined maximum. Until 2010, matching programs were used in a number of jurisdictions, most prominent among them Arizona and Maine. In 2011, the Supreme Court declared these programs unconstitutional, as matching mechanisms disburse public funds in direct response to private campaign spending, impermissibly burdening the speech of privately financed candidates (*Arizona Free Enterprise Club's Freedom Club PAC v. Bennett*, 564 U.S. __ 2011, henceforth "Arizona Free Enterprise").[3]

3. Arizona and Maine enacted clean elections acts in 1998 and 1996, respectively, and used matching mechanisms comprehensively in all elections for state offices. In addition, several other states have used matching provisions as part of their public election funding programs. Minnesota operated an early matching program but was forced to abolish it in 1994 following a federal court decision (*Day v. Holahan*, 34 F.3d 1356, U.S. Court of Appeals, 8th Circuit, 1994). Connecticut adopted its matching program in 2006, but abolished it 4 years later following a separate federal court decision (*Green Party of Connecticut v. Garfield*, 616 F.3d 189, U.S. Court of Appeals, 2nd Circuit, 2010). Matching programs were also used in judicial elections in North Carolina and West Virginia, gubernatorial elections in Florida, as well as in municipal elections in Albuquerque, Los Angeles, and San Francisco. For detailed legal analyses of matching provisions in public campaign finance and the court challenges against them, see LoBiondo (2011), Hudson (2012), Rahmanpour (2012), and Steele (2012).

We examine both funding mechanisms in our contest model and establish a number of surprising, and often counterintuitive, results. These results call into question the way in which the Supreme Court has applied its "leveling the playing field" doctrine to determine the constitutionality of public campaign funding programs. Specifically, we reach the following conclusions:

1. We show that a public option with matching funds is equivalent, in terms of the candidates' participation decisions, equilibrium election probabilities, private spending, and payoffs, to a simple public option whose lump-sum transfer equals the maximum possible funding level under the matching program. Thus, the argument that the provision of matching funds, in order to level the playing field, burdens private speech would, if valid, apply equally to simple public options. Yet, simple public options remain legal while those that include a matching mechanism do not.
2. We then examine if public funding levels the political playing field in the first place. We demonstrate that, when some candidates choose to participate in the public program but others do not, privately funded candidates may be more *or* less likely to win than they would be if public funding were not available to their opponents. Conversely, removing or restricting a public financing program may increase *or* decrease the election probability of candidates that do not accept the public option. The reason is that a publicly funded candidate is constrained by the maximum funding level permitted under the program, and thus can be outspent by an unconstrained candidate relatively easily. It is even possible that all candidates prefer the availability of a public option over a purely private system of campaign finance, including candidates who do not use the public option.[4] Yet, the plight of privately funded candidates who ran against state-funded opponents was a major concern of the Supreme Court in *Arizona Free Enterprise*.

4. This scenario can indeed be an equilibrium: Financially weaker candidates may prefer to accept public funding, not because of how it affects their probability of election but because of the cost-savings it entails. Conversely, financially stronger candidates may prefer to run against publicly funded candidates who, because they participate in the public program, are constrained in their private fundraising and spending activities.

3. Finally, we examine how public campaign financing affects the candidates' incentives to engage in political speech. We show that if a small increase in the funding level of a public program induces additional candidates to forgo private fundraising and receive state funds instead, political speech decreases. However, this effect is reversed when participation decisions do not change. In particular, we show that in any equilibrium in which one candidate accepts a simple public option while the other elects to raise funds privately, the introduction of additional matching funds increases speech by *all* candidates. We also show that public funding systems that increase total speech are systems that level the playing field.

In addition to casting doubt on the validity of several of the arguments the Supreme Court made when evaluating the effects of public campaign funding programs, our theoretical results also allow us to examine the validity of empirical assessments of these effects. Some authors have proposed the following test to determine whether a public option with matching funds chills political speech: If it does, then private campaign spending should cluster just below the initial disbursement paid to publicly funded candidates (Gierzynski, 2011; Dowling et al., 2012). Election finance data for states that had matching programs does not reveal such clustering, suggesting that privately funded candidates were not effectively constrained in their speech. The state of Arizona used the same argument when it defended its matching provision before the Supreme Court. We show that the presence or absence of clustering cannot be used to infer that public funding affects the level of speech.

In sum, our results suggest that a number of important strategic aspects of public campaign funding programs may have been misunderstood by courts and academics alike. The game-theoretic model that leads us to this conclusion is admittedly stylized. However, the fact that the predictions from even this simple model are often at odds with intuition provides all the more reason to be cautious when using intuitive reasoning to predict the effects of public funding programs on election outcomes, campaign spending, and political speech.

The remainder of the paper is organized as follows. In Section 2, we review the related literature. In Section 3, we present a contest model of two-candidate elections with private and public campaign funding, and in Section 4 we derive its equilibria. In Section 5, we discuss the Supreme Court's *Arizona Free Enterprise* ruling in light of our model results. In Section 6, we examine the extent to which our results are robust to a number of alternative modeling choices. These include funding decisions that are made sequentially (instead of simultaneously), asymmetries in the candidates' impact of political speech (instead of their fundraising costs), and the introduction of risk in private fundraising. Section 7 concludes. Most proofs are in the Appendix.

2. Literature Review

The clean elections acts passed in Arizona and Maine in the late 1990s were the first clean elections acts and still constitute the most comprehensive attempts at campaign finance reform in the United States to date (for an overview of earlier reforms, see Jones, 1981). For this reason, a number of studies have examined the effects of public campaign financing in both states. These papers investigate a very similar set of questions that we examine here: How does public funding affect candidates' election probabilities; how does public funding affect campaign spending and political speech; and what are the effects (if any) of providing public funds through matching mechanisms instead of lump-sum. Unlike our paper, however, the previous literature has addressed these questions empirically.

Public campaign financing and election probabilities. Malhotra (2008) compares win margins for incumbents in senate races in Arizona and Maine before and after these states introduced a public option in 2000. He finds that win margins for incumbents decreased from 1998 to 2000. Stratmann (2009) finds that public financing in Maine reduced vote margins in House elections compared with other states with limited or no public financing. However, in a recent survey of the literature on the effects of laws on public funding of elections across all states, Mayer (2013) concludes that while there is some evidence that public funding may have slightly increased competitiveness in state legislative elections, mainly by reducing the number of uncontested elections, there is no evidence that it has increased competitiveness of contested elections. Moreover, there is no evidence that it has changed incumbency reelection rates or margins of victory in the longer run. The effects are even less discernable in statewide elections. Primo et al. (2006) find no significant effect of public funding on the competitiveness of gubernatorial elections.

Public campaign financing and election spending. Miller (2011a) finds that total spending in Arizona House and Senate races increased significantly after 2000, and that total spending in Maine House and Senate races decreased slightly after 2000. Miller (2011b) finds that publicly funded candidates in Arizona and Maine spent more time interacting with the public in crucial election phases, compared with privately funded candidates (a possible reason being that publicly funded candidates need to spend less time fundraising). Miller (2012) finds that the introduction of a public option in Arizona and Maine yielded greater benefits to Democratic challengers than to Republican ones.

Public campaign financing and matching programs. Several papers examine specifically the matching mechanisms that were part of the programs operated by Arizona and Maine. Miller (2008) and United States Government Accountability Office (2010) examine empirically the effects of matching funds provisions in Arizona and Maine on the timing of privately funded candidates' expenditures. These studies find that privately funded candidates strategically delay their expenditures in order to postpone triggering matching funds for their publicly funded opponents. Gierzynski (2011) and Dowling et al. (2012) examine whether contributions to privately financed candidates in state congressional elections in Maine and Arizona exhibit clustering below the initial funding level of publicly financed candidates. Since contributions beyond this threshold trigger matching payments to publicly financed candidates, clustering just below the matching threshold might indicate that matching funds chill private speech. However, no evidence of such clustering is found. Dowling et al. (2012) also compare the evolution of aggregate campaign contributions in Arizona to those in Maine, as well as a synthetic control state, to estimate the treatment effect associated with the injunction halting Arizona's matching program in 2010. No evidence is found that the injunction increased contributions in Arizona, relative to the comparison states.

Our paper contributes to this literature by providing a formal, game-theoretic model that allows us to examine the channels through which both simple public options and matching programs affect election probabilities and campaign spending. It complements the empirical literature by providing an analytical framework, based on contest theory, that can help explain several of the empirical findings discussed above. For example, we provide conditions under which public campaign funding makes elections more competitive and less competitive, and we explain *why* public funding programs that include a matching component do not result in clustered spending by privately funded candidates.

Finally, we note for completeness that a number of authors have investigated other types of matching mechanisms. Coate (2004) and Ashworth (2006) theoretically analyze the properties of campaign finance systems in which the state matches any private contributions to a party with public contributions to the same party.[5] These programs can provide a "continuous" alternative to more conventional *threshold grants* that require candidates to demonstrate their viability by collecting a certain amount of private donations before becoming eligible for a fixed public subsidy. Our paper does not analyze this type of matching mechanism. In contrast, we examine a system that matches private contributions to a candidate with public contributions to opposing candidates. Ortuno-Ortín and Schultz (2004), Prat et al. (2010), and Klumpp (2014) examine European systems of public financing, in which candidates receive public funds in proportion to their vote shares. In contrast, we analyze American systems of public financing, in which candidates receive a lump-sum transfer of money and, possibly, matching grants that depend directly on contributions to their opponents, but not on election outcomes.

3. Election Contests with Public Campaign Funding

Two candidates ($i = 1, 2$) compete to win an election. Both candidates attach a value of one to winning the election, and a value of zero to losing. We model electoral competition as a speech (or advertising) contest. Let $x_i \geqslant 0$ denote candidate i's amount of speech ($i = 1, 2$). The probability

5. Such programs are presently in use in Florida, Hawaii, Maryland, Massachusetts, Michigan, New Jersey, and Rhode Island. Source: National Conference of State Legislatures (www.ncsl.org).

that candidate i wins the election is given by the *Tullock contest success function*:

$$P_i = f(x_i, x_{-i}) \equiv \begin{cases} \dfrac{x_i}{x_i + x_{-i}} & \text{if } x_1 + x_2 > 0, \\ \dfrac{1}{2} & \text{if } x_1 + x_2 = 0, \end{cases} \tag{1}$$

where, as usual, $-i$ denotes i's opponent. The success function in (1), introduced in Tullock (1980), is one of several "workhorse models" employed in the literature on contests and rent-seeking (see Konrad, 2009 for an overview).

Each unit of advertising costs one unit of money and has to be paid by the politician's campaign. For this, the campaign needs to raise contributions, either from private sources or from public sources. These funding sources are modeled as follows.

Private funding. If candidate i is privately funded, he freely chooses x_i. The cost of raising these private funds is $c_i x_i$. The coefficient $c_i \in (0, \infty)$ has several interpretations. It may represent i's opportunity cost of a dollar spent on campaigning. A candidate with a low c_i can be thought of as a "rich" politician, and a candidate with a high c_i can be thought of as a "poor" politician. Alternatively, c_i may represent i's fundraising costs. A candidate with a low c_i can be thought of as a politician with access to a large number of wealthy supporters, relative to one with a high c_i. Both candidates' fundraising cost parameters, c_1 and c_2, are common knowledge.[6]

Public funding. A candidate who participates in a public program incurs no fundraising costs; however, this candidate cannot raise or spend private funds. Instead, publicly funded candidate i receives an initial transfer $T_0 \geqslant 0$ from the state to spend on his campaign.

A participating candidate whose opponent is privately funded may receive additional transfers, which are determined as follows: Let i be the publicly funded candidate and let $-i$ be i's privately funded opponent. For every dollar spent by $-i$ above T_0, candidate i receives one dollar from the state, until i's funding reaches a maximum $T_{\max} \geqslant T_0$. The overall funds

6. In Section 6.3, we discuss other characteristics in which candidates could differ.

received by the publicly financed candidate are then given by the formula

$$x_i = \gamma(x_{-i}) \equiv \begin{cases} T_0 & \text{if } x_{-i} \leqslant T_0, \\ x_{-i} & \text{if } T_0 < x_{-i} < T_{\max}, \\ T_{\max} & \text{if } x_{-i} \geqslant T_{\max}. \end{cases} \tag{2}$$

If both candidates are publicly funded, then both receive only the initial transfer T_0.[7]

A public funding program is hence a pair $(T_0, T_{\max}) \in [0, \infty)^2$, consisting of the minimum and maximum funding level of participating candidates. A *simple public option* is a program such that $0 < T_0 = T_{\max}$. If $T_{\max} > T_0$, we say that the program has *matching funds*. A program $0 = T_0 < T_{\max}$ has only matching funds, and the program $(0, 0)$ is equivalent to there being no public campaign funding.

The timing of the game is as follows. First, both candidates simultaneously decide whether to participate in the public funding program or not (stage 1). For candidate $i = 1, 2$, this choice is denoted $s_i \in \{\text{Pr}, \text{Pu}\}$. The candidates then observe each other's choice and engage in the electoral contest (stage 2). A publicly funded candidate does not have any further decisions to make, as his spending level is determined automatically via (2). A privately funded candidate, on the other hand, has to decide how much to spend.

We will investigate the subgame perfect equilibria of the game described above. These can be found as follows. Let $v_i(\cdot|s_i, s_{-i})$ be candidate i's payoff function in the electoral contest at stage 2, given the funding choices s_i

7. The actual matching programs that were used in Arizona and Maine did not match dollar-for-dollar but, instead, contained small reductions in the matching rate that were meant to offset private fundraising expenses. For example, Arizona's 2008 funding formula awarded to every participant who ran as a candidate for state representative in district d the amount

$$\gamma(\overline{x}_d) = \$21,479 + \min\{0.94 \cdot \max\{0, \overline{x}_d - \$21,479\}, \$42,958\},$$

where $\overline{x}_d$ denotes the maximum private spending among the candidates in district d. Furthermore, both the matching rate of 94 cents per dollar and the maximum public funding level \$42,958 could be adjusted by the state election commission at various points during an election cycle, depending on the proportion of Arizona's budgeted public campaign funds that had already been allocated. We abstract from these details here.

and s_{-i}. This payoff function is

$$v_i(x_i, x_{-i} \mid s_i, s_{-i}) = \begin{cases} f(x_i, x_{-i}) - c_i x_i & \text{if } (s_i, s_{-i}) = (\text{Pr, Pr}), \\ f(x_i, \gamma(x_i)) - c_i x_i & \text{if } (s_i, s_{-i}) = (\text{Pr, Pu}), \\ f(\gamma(x_{-i}), x_{-i}) & \text{if } (s_i, s_{-i}) = (\text{Pu, Pr}), \\ f(T_0, T_0) & \text{if } (s_i, s_{-i}) = (\text{Pu, Pu}). \end{cases} \tag{3}$$

Let $v_i^*(s_1, s_2)$ be candidate i's expected Nash equilibrium payoff in the subgame at (s_1, s_2). At the initial stage 1 of our model, a pure strategy Nash equilibrium in funding choices is then a pair $(s_1^*, s_2^*) \in \{\text{Pu, Pr}\} \times \{\text{Pu, Pr}\}$ such that

$$v_i^*(s_i^*, s_{-i}^*) \geqslant v_i^*(s_i, s_{-i}^*) \quad \text{for } i = 1, 2 \quad \text{and } s_i = \text{Pu, Pr}.$$

In the next section, we find the subgame perfect equilibria by first computing the second-stage payoffs v_1^* and v_2^* and then finding a stage 1 equilibrium in funding choices (s_1^*, s_2^*).

Before we go on, let us note that for the purpose of this paper we equate political speech, advertising, campaign spending, and campaign fundraising. Each of these assumed equivalences is a simplification. First, some forms of political speech do not require much in the way of costly advertising (e.g., a politician giving a media interview), and we ignore such types of "non-costly" speech in our model. Secondly, campaigns routinely spend money they have raised on activities other than advertising (e.g., staff salaries, office rent, travel, or voter mobilization). Our model also does not distinguish these non-advertising expenses as a separate spending category. However, to the extent that they support the communication of the candidate's message, or increase the candidate's chance of success in some other way, they fit the formal structure of our contest model. Thirdly, campaigns sometimes raise more money than they spend (allowing a politician to save resources for a future election), and sometimes they spend more money than they raise (in which case the campaign must repay its debt after the election). Because ours is a static, one-shot model, we do not consider these possibilities. Instead, we assume that a campaign spends precisely as much money as it raises. These simplifications allow us to focus on the issues raised by the Supreme Court when it evaluated public campaign funding programs in light of the First Amendment.

4. Equilibrium Characterization

In this section, we characterize the equilibrium funding choices of the candidates, their advertising levels, their success probabilities, and their payoffs. In the spirit of backward induction, we first solve the second-stage contest conditional on the candidates' funding choices, and then use these results to determine the equilibrium funding choices at the first stage.

4.1. Analysis of the Second-Stage Contest

Consider a public campaign funding program $(T_0, T_{\max})$. Suppose each candidate has made his decision of whether to participate in this program or not. The following three cases can then arise at the second stage of the game.

First, if both candidates are publicly funded the outcome is trivial. Both candidates receive from the state the transfer T_0, which they spend in the contest at no personal cost to themselves. Therefore, we have $x_1 = x_2 = T_0$ and $P_1 = P_2 = 1/2$, and the candidates' payoffs in this subgame are

$$v_1^*(\text{Pu}, \text{Pu}) = v_2^*(\text{Pu}, \text{Pu}) = \tfrac{1}{2}.$$

Next, if both candidates are privately funded, stage 2 of our model becomes a standard Tullock contest with common prize 1 and marginal costs c_1 and c_2. This contest has a well known, unique pure strategy Nash equilibrium, in which spending levels and success probabilities are

$$x_1 = \frac{c_2}{(c_1 + c_2)^2}, \quad x_2 = \frac{c_1}{(c_1 + c_2)^2}, \quad \text{and}$$
$$P_1 = \frac{c_2}{c_1 + c_2}, \quad P_2 = \frac{c_1}{c_1 + c_2}.$$

The payoffs in this equilibrium are given by

$$v_1^*(\text{Pr}, \text{Pr}) = \frac{(c_2)^2}{(c_1 + c_2)^2}, \quad v_2^*(\text{Pr}, \text{Pr}) = \frac{(c_1)^2}{(c_1 + c_2)^2}.$$

For details on the computation, we refer the reader to the large literature on Tullock contests (see, for example, Konrad, 2009).

Finally, consider the case where one candidate is privately funded and the other is publicly funded. Without loss of generality, assume that candidate 1 is privately funded and candidate 2 is publicly funded (the analysis is similar when the roles are reversed). Candidate 1's payoff when spending x_1 against his opponent in the public funding system $(T_0, T_{\max})$ is

$$\frac{x_1}{x_1+\gamma(x_1)} - c_1x_1 = \begin{cases} \dfrac{x_1}{x_1+T_0} - c_1x_1 & \text{if } x_1 \leqslant T_0, \\ \dfrac{1}{2} - c_1x_1 & \text{if } T_0 < x_1 < T_{\max}, \\ \dfrac{x_1}{x_1+T_{\max}} - c_1x_1 & \text{if } x_1 \geqslant T_{\max}. \end{cases} \tag{4}$$

Note that, in case of a simple public option, only the first line on the right-hand side of (4) is relevant. Maximizing this payoff function with respect to x_1, we obtain

$$x_1 = \begin{cases} 0 & \text{if } c_1 > \dfrac{1}{T_0}, \\ \sqrt{T_0/c_1} - T_0 & \text{if } \dfrac{1}{T_0} \geqslant c_1 > \dfrac{1}{4T_0}, \\ T_0 & \text{if } \dfrac{1}{4T_0} \geqslant c_1 > \dfrac{1}{4T_{\max}}, \\ \sqrt{T_{\max}/c_1} - T_{\max} & \text{if } c_1 \leqslant \dfrac{1}{4T_{\max}}. \end{cases} \tag{5}$$

(Again, in case of a simple public option, the two middle cases in (5) collapse into one.) The money received by publicly funded candidate 2 is now determined by using (5) in the funding formula (2):

$$x_2 = \gamma(x_1) = \begin{cases} T_0 & \text{if } c_1 > \dfrac{1}{4T_{\max}}, \\ T_{\max} & \text{if } c_1 \leqslant \dfrac{1}{4T_{\max}}. \end{cases} \tag{6}$$

Plugging both (5) and (6) into the Tullock success function, we get the following success probabilities for candidate 1:

$$P_1 = f(x_1, x_2) = \begin{cases} 0 & \text{if } c_1 > \dfrac{1}{T_0}, \\ 1 - \sqrt{c_1 T_0} & \text{if } \dfrac{1}{T_0} \geqslant c_1 > \dfrac{1}{4T_0}, \\ \dfrac{1}{2} & \text{if } \dfrac{1}{4T_0} \geqslant c_1 > \dfrac{1}{4T_{\max}}, \\ 1 - \sqrt{c_1 T_{\max}} & \text{if } c_1 \leqslant \dfrac{1}{4T_{\max}}. \end{cases} \tag{7}$$

Candidate 2's success probability is then $P_2 = 1 - P_1$, and because candidate 2 has no campaign costs his payoff is also P_2. Candidate 1's payoff is his election probability, P_1, minus his private cost, $x_1 c_1$. After a few manipulations, this payoff can be written as

$$v_1^*(\text{Pr}, \text{Pu}) = \begin{cases} 0 & \text{if } c_1 > \dfrac{1}{T_0}, \\ \left(1 - \sqrt{c_1 T_0}\right)^2 & \text{if } \dfrac{1}{T_0} \geqslant c_1 > \dfrac{1}{4T_0}, \\ \dfrac{1}{2} - c_1 T_0 & \text{if } \dfrac{1}{4T_0} \geqslant c_1 > \dfrac{1}{4T_{\max}}, \\ \left(1 - \sqrt{c_1 T_{\max}}\right)^2 & \text{if } c_1 \leqslant \dfrac{1}{4T_{\max}}. \end{cases} \tag{8}$$

4.2. Analysis of the First-Stage Funding Choices

The second-stage continuation payoffs $v_1^*(s_1, s_2)$ and $v_2^*(s_1, s_2)$ that we derived in Section 4.1 define a normal form game that describes the initial stage of our game, at which candidates choose whether or not to participate in the public campaign funding program (i.e., they choose s_1 and s_2). In an overall subgame perfect equilibrium of our model, the stage-1 funding choices must form a Nash equilibrium. We distinguish three types of pure strategy equilibrium:

1. *All-public* equilibrium: Both candidates accept the public option, i.e., $(s_1, s_2) = (\text{Pu}, \text{Pu})$.

2. *All-private* equilibrium: Both candidates choose private funding, i.e., $(s_1, s_2) = (\mathrm{Pr}, \mathrm{Pr})$.
3. *Private–public* equilibrium: One candidate chooses private funding while the other chooses public funding, i.e., $(s_1, s_2) = (\mathrm{Pr}, \mathrm{Pu})$ or $(s_1, s_2) = (\mathrm{Pu}, \mathrm{Pr})$.

Depending on the parameters of the model—that is, the fundraising costs c_1 and c_2 and the policy parameters T_0 and $T_{\max}$—all three equilibrium types can emerge:

PROPOSITION 1 The following characterizes all pure strategy equilibria of the model. Define

$$K \equiv \frac{3/2 - \sqrt{2}}{T_{\max}}, \quad L(c) \equiv c \cdot ((cT_{\max})^{-1/4} - 1). \tag{9}$$

(a) An all-public equilibrium exists if and only if $c_i \geqslant K$ $(i = 1, 2)$. In this equilibrium, both candidates spend T_0 from state funds and both win with probability $1/2$.

(b) An all-private equilibrium exists if and only if $c_i \leqslant L(c_{-i})$ $(i = 1, 2)$. In this equilibrium, candidate i spends $c_{-i}/(c_1 + c_2)^2$ from private funds and wins with probability $c_{-i}/(c_1 + c_2)$.

(c) A private–public equilibrium in which candidate i is privately funded and candidate $-i$ accepts the public option exists if and only if $c_i \leqslant K$ and $c_{-i} \geqslant L(c_i)$. In this equilibrium, candidate i spends $\sqrt{T_{\max}/c_i} - T_{\max}$ from private funds and candidate $-i$ spends $T_{\max}$ from public funds. Privately funded candidate i wins with probability $1 - \sqrt{c_i T_{\max}}$, and publicly funded candidate $-i$ wins with probability $\sqrt{c_i T_{\max}}$.

Figure 1 depicts the parameter regions that give rise to the equilibria identified in Proposition 1. If both candidates have relatively high fundraising costs, they both accept the public option (the top right corner of the figure). If both have relatively low costs, they both reject the public option (the lense-shaped region in the bottom left corner). If the candidates' fundraising costs are sufficiently asymmetric, the high-cost candidate chooses public financing while the low-cost candidate remains

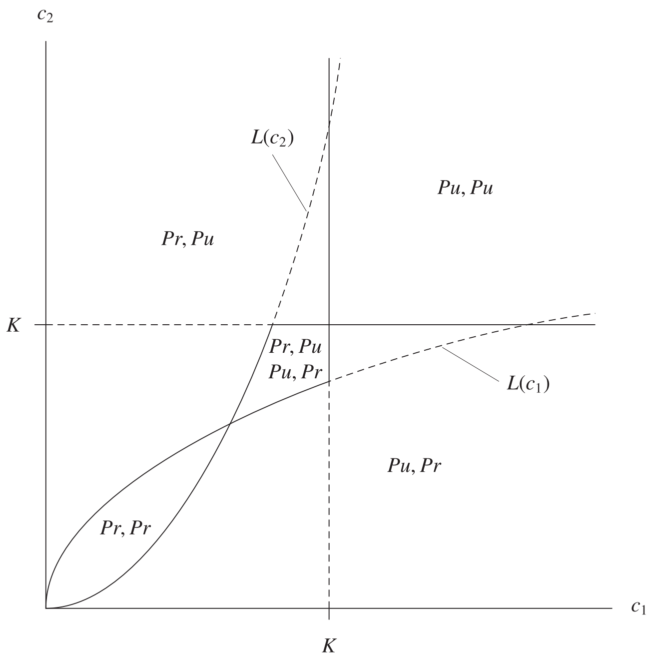

Figure 1. Equilibrium Funding Choices for Different c_1, c_2-Combinations.

privately funded. In the bottom right region, candidate 1 chooses the public option; and in the top left region, candidate 2 chooses the public option.

In the small diamond-shaped area at the center of the figure, two types of private–public equilibrium exist—one in which candidate 1 accepts the public option but not candidate 2, and one in which the roles are reversed. In this area, which is characterized by the inequality $L(c_{-i}) < c_i < K$ $(i = 1, 2)$, the candidates' first-stage problem constitutes an anti-coordination game (a game of "Chicken"). This implies that there is also a mixed strategy equilibrium in which candidates randomize over public and private financing.[8]

8. In general, a Nash equilibrium (in pure or mixed strategies) is a pair $(p_1, p_2) \in [0, 1] \times [0, 1]$, where p_i is the probability that candidate i participates in the public program, such that $p_i > 0$ $(p_i < 1)$ implies

$$p_{-i} v_i(\text{Pu, Pu}) + (1 - p_{-i}) v_i^*(\text{Pu, Pr}) \geqslant (\leqslant) \quad p_{-i} v_i(\text{Pr, Pu}) + (1 - p_{-i}) v_i^*(\text{Pr, Pr}) \quad (i = 1, 2).$$

5. Putting the Model to Use: A Rebuttal to *Arizona Free Enterprize*

In *Arizona Free Enterprise*, the Supreme Court ruled that the clean election program operated by the state of Arizona prior to 2010, whose central component was a public option with matching funds, was unconstitutional. The Court's decision was based on the following line of reasoning:

1. Public options with matching grants burden political speech in ways simple public options do not.
2. The purpose of imposing this burden is to "level the political playing field," which is not a compelling state interest in regulating speech.
3. The effect of a more level playing field is a decrease in the political speech of at least some candidate.

The equilibria we characterized in the previous section have interesting properties that bear directly on each of these arguments. We consider them in turn to assess the validity of the Court's reasoning within the context of our model. In Section 5.1, we examine whether there is a material difference between simple public options and matching programs; in Section 5.2, we examine whether public campaign funding (with and without matching funds) levels the political playing field; and in Section 5.3 we examine whether a more level playing field chills political speech.

5.1. Are Matching Grants and Simple Options Really Different?

The Supreme Court's central concern in *Arizona Free Enterprise* was not the public funding of candidates *per se*, but the matching mechanism through which the state allocated public funds to candidates. In particular, the Court suggested that even a large lump-sum transfer from the state to

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If a mixed strategy equilibrium exists, the probability that candidate i chooses the public option can be shown to be

$$p_i = \left(\sqrt{c_i T_{\max}} - \frac{c_i^2}{(c_1 + c_2)^2}\right) \Bigg/ \left(\sqrt{c_i T_{\max}} - \frac{c_i^2}{(c_1 + c_2)^2} + \left(1 - \sqrt{c_{-i} T_{\max}}\right)^2 - \frac{1}{2}\right).$$

We will not consider the possibility of mixed strategy equilibrium in the rest of our analysis.

candidates may be constitutional because it does not depend on the political speech of any privately financed candidates:

The State correctly asserts that the candidates and independent expenditure groups 'do not [...] claim that a single lump sum payment to publicly funded candidates,' equivalent to the maximum amount of state financing that a candidate can obtain through matching funds, would impermissibly burden their speech. . . . The State reasons that if providing all the money up front would not burden speech, providing it piecemeal does not do so either. And the State further argues that such incremental administration is necessary to ensure that public funding is not under- or over-distributed. . . . These arguments miss the point. It is not the amount of funding that the State provides to publicly financed candidates that is constitutionally problematic in this case. It is the manner in which that funding is provided—in direct response to the political speech of privately financed candidates and independent expenditure groups. (564 U.S. __ 2011, at 21.)

We now examine if this distinction between lump-sum transfers and matching grants is of any consequence in our model. Proposition 1 shows that the candidates' incentives to select public or private financing depends on the public funding program $(T_0, T_{\max})$ only through the maximal level of state funding $T_{\max}$, but not on T_0. Moreover, in every equilibrium the candidates' election probabilities depend on $T_{\max}$ only, and the same is true for the fundraising and spending of private candidates. Hence, we have the following.

Corollary 1 Financing decisions, election probabilities, private spending, and the candidates' payoffs under any funding program $(T_0, T_{\max})$ that includes matching funds are the same as those under an alternative program $(T_{\max}, T_{\max})$ that consists only of a simple public option in the amount $T_{\max}$ but no matching funds.

The intuition behind Corollary 1 is simple. Public programs with the same maximum funding level $T_{\max}$ result in different outcomes only when candidates who reject public funding spend $<T_{\max}$. But a candidate should not reject state funding unless he is prepared to spend *more* than $T_{\max}$: Spending $<T_{\max}$ from private funds against a publicly funded opponent is costly and results in a probability of election that is at most one-half. Choosing public funding against the same opponent, on the other hand, entails

no costs and guarantees an election probability of one-half. Thus, a case in which the election probability of a privately funded candidate under a matching program $(T_0, T_{\max})$ is different from that under a simple public option of value $T_{\max}$ cannot arise in equilibrium.[9]

Note that Corollary 1 does not say that removing a matching component from a public program will not change election outcomes or speech. What it says is that any public funding program that includes a matching component is equivalent—in terms of program participation, election outcomes, and private speech—to a simple public option whose value is equal to the maximal state funding level under the matching program. An implication of this result is that states can undo restrictions on matching programs imposed by courts by adopting an appropriately chosen simple public option that replicates the equilibrium outcomes under the matching program. For example, this is how Connecticut adjusted its public funding program for gubernatorial elections when its matching funds provision was ruled unconstitutional by a federal court in 2010 (Thomas, 2010).

Moreover, Proposition 1 implies that this adjustment also leaves public campaign spending unchanged except when *all* candidates choose public funding, in which case public spending is higher in program $(T_{\max}, T_{\max})$ than in program $(T_0, T_{\max})$. Therefore, on expectation, a matching program costs the state less to operate than a simple option that results in the same participation incentives, the same amount of privately funded speech, and the same election outcomes.[10] Note that states that operated a matching program $(T_0, T_{\max})$ and do not have the resources to offer the public option $(T_{\max}, T_{\max})$ could respond by replacing their matching program with a simple public option (T, T), where $T_0 < T < T_{\max}$. In this case, the candidates' participation decisions as well as their speech may be affected. In particular, a candidate who would have accepted the public option in programs $(T_0, T_{\max})$ and $(T_{\max}, T_{\max})$ may decline it in program (T, T). In Proposition 4 in Section 5.3, we show that, if candidates adjust their participation

9. We will return to this dominance argument again at the end of Section 5.3, where we find that the Court applied it *correctly* when it rejected empirical evidence presented by Arizona in defense of its matching program, and in Section 6.1, where we examine its robustness to fundraising uncertainty as well as alternative candidate payoff functions.

10. The same argument was made by Judge Elena Kagan in her dissent to *Arizona Free Enterprise*. See 564 U.S. __ 2011, Kagan, J., dissenting, at 9.

decision and the change in the program funding level is not too drastic, both candidates' speech increases (while the cost to the state decreases).

It follows that, in our model, a matching program does not impose a burden on private political speech that would not be the same in a program that consisted only of an "equilibrium-equivalent" simple public option. (In fact, the burden imposed by a simple public option could even be more severe, as we will argue in Section 6.1.) Thus, if public options with matching funds are unconstitutional because they burden the speech of privately funded candidates or independent expenditure groups, then the same must be true for simple options.

5.2. Does Public Campaign Funding Level the Political Playing Field?

In a series of First Amendment cases, the U.S. Supreme Court has ruled that burdens on political speech that are imposed to achieve a balance of speech between candidates are unconstitutional (see Footnote 3). The decision in *Arizona Free Enterprise* follows the same doctrine:

> We have repeatedly rejected the argument that the government has a compelling state interest in 'leveling the playing field' that can justify undue burdens on political speech. . . . [I]n a democracy, campaigning for office is not a game. It is a critically important form of speech. The First Amendment embodies our choice as a Nation that, when it comes to such speech, the guiding principle is freedom—the 'unfettered interchange of ideas'—not whatever the State may view as fair. (564 U.S. __ 2011, at 24–25.)

In the preceding section, we compared public financing programs that include matching grants and those that consist only of a simple option equal to the maximum funding level under the matching program. We showed that both result in the same relative speech by each candidate, and hence in the same election probabilities. Put differently, in our model matching grants and simple options alter the balance of political speech in the same way, compared with a hypothetical scenario in which no public funding is available. Thus, if matching programs are unconstitutional because states may use them to "level the playing field," the same must be true for simple public options.

The Court's reasoning when it declared matching funds unconstitutional is based on the assumption that a public option with matching funds does, indeed, "level the playing field," and does so at the expense of burdening political speech. In this section, we examine *how* public funding programs—with or without matching funds—affect the balance of political speech, showing that it can increase as well as decrease balance. In the following section, we examine how it affects the quantity of speech, showing that it can increase as well as decrease speech, and that it tends to increase speech in cases where it "levels the playing field." To do so, we compare our model equilibria under a public financing program to those of a counterfactual contest in which no public funding is available. The equilibrium of this counterfactual contest will necessarily be of the all-private type. As shown in Section 4.1, in an all-private equilibrium the candidate with the larger fundraising cost (the *disadvantaged candidate*) is outspent by the candidate with the lower fundraising cost (the *advantaged candidate*) and is less likely to win against this opponent. Compared with this case, a public financing program can have five possible effects on relative spending and election probabilities:

1. it *unlevels* the playing field if the disadvantaged candidate's equilibrium funding share and election probability is less than it would be without the program.
2. It leaves the playing field unchanged if the disadvantaged candidate's equilibrium funding share and election probability is the same as it would be without the program.
3. It *partially levels* the playing field if the disadvantaged candidate's equilibrium funding share and election probability is greater than it would be without the program, but $<1/2$.
4. It *fully levels* the playing field if the disadvantaged candidate's equilibrium funding share and election probability is equal to $1/2$.
5. It *reverses* the playing field if the disadvantaged candidate's equilibrium funding share and election probability is $>1/2$.

All five possibilities can arise in our model, and the following result provides a complete characterization of the possible effects of public financing on the political playing field, assuming candidates play pure strategy equilibria:

PROPOSITION 2 Suppose that candidate i is the advantaged candidate (i.e., $c_i < c_{-i}$). The effects of public funding program $(T_0, T_{\max})$ in pure strategy equilibrium are the following:

(a) The program unlevels the playing field if and only if

$$\left(\frac{c_i}{c_1+c_2}\right)^2 \frac{c_i}{(c_1+c_2)^2} \leqslant T_{\max} < \min\left\{\frac{c_i}{(c_1+c_2)^2}, \frac{3/2-\sqrt{2}}{c_i}\right\}.$$

(b) The program leaves the playing field unchanged if and only if

$$T_{\max} \leqslant \left(\frac{c_i}{c_1+c_2}\right)^2 \frac{c_i}{(c_1+c_2)^2}.$$

(c) The program partially levels the playing field if and only if

$$\frac{c_i}{(c_1+c_2)^2} < T_{\max} \leqslant \frac{3/2-\sqrt{2}}{c_i}.$$

(d) The program fully levels the playing field if and only if

$$T_{\max} \geqslant \frac{3/2-\sqrt{2}}{c_i}.$$

(e) The program reverses the playing field if and only if

$$\left(\frac{c_{-i}}{c_1+c_2}\right)^2 \frac{c_{-i}}{(c_1+c_2)^2} \leqslant T_{\max} \leqslant \frac{3/2-\sqrt{2}}{c_{-i}}.$$

All five cases in Proposition 2 depend on the candidates' costs in relation to the maximum public funding level $T_{\max}$, but not on T_0 (showing again that it does not matter if $T_{\max}$ is distributed as a lump-sum payment or through a matching mechanism). Figure 2 depicts these cases for different (c_1, c_2)-pairs, holding $T_{\max}$ fixed.

The "full leveling" and "no change" regions correspond precisely to the all-public and all-private regions in Figure 1. The private–public region is divided into a part where fundraising costs are relatively asymmetric, resulting in a partial leveling of the playing field; and a part where fundraising costs are relatively symmetric, resulting in an unleveling of the playing field. The possibility of a reversed playing field arises in the small center

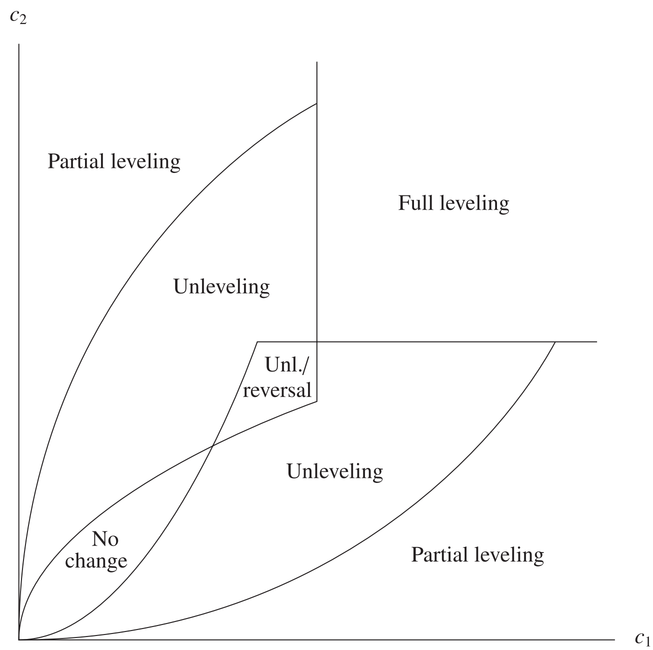

Figure 2. The Effect of Public Funding on Relative Spending and Election Probabilities.

region where our model gave rise to two private–public equilibria. A reversal occurs in the equilibrium in which the candidate with the higher cost rejects public financing, while the candidate with the lower cost accepts it. In this case, the advantaged candidate would have been more likely to win than his opponent if both were forced to raise private funds, but becomes less likely to win once he accepts public funding. Despite the smaller chance of victory, however, accepting public funding is optimal because it eliminates the candidate's fundraising costs.

The fact that a candidate may want to accept public funding even if this reduces his chance of victory has an interesting "twin" property: A candidate who does *not* participate in the public program may benefit from its presence (and from the fact that it finances the opponent's campaign), because it increases his chance of victory. Consider, for example, candidates with costs $c_1 = 0.1$ and $c_2 = 0.15$. Without public funding, the candidates' speech, election probabilities, and payoffs in an all-private equilibrium are as follows:

No public funding:

$$x_1 = 2.4, \quad x_2 = 1.6, \quad P_1 = 0.6, \quad P_2 = 0.4,$$
$$v_1 = 0.36, \quad v_2 = 0.16.$$

If a public funding program with $T_{\max} = 0.5$ is available, then there will be a private–public equilibrium in which candidate 1 is privately funded and candidate 2 is publicly funded. In this equilibrium, we have

Public funding ($T_{\max} = 0.5$):

$$x_1 = 1.736, \quad x_2 = 0.5, \quad P_1 = 0.776, \quad P_2 = 0.224,$$
$$v_1 = 0.603, \quad v_2 = 0.224.$$

Here, public financing unlevels the playing field and at the same time decreases the absolute amount of speech by each candidate. Both effects help the privately funded candidate, who is now more likely to win with less effort. The disadvantaged candidate is less likely to win, but prefers this outcome all the same because of the cost-savings he enjoys by not having to raise private funds. In this example, the availability of public funding is a Pareto improvement for the candidates even though only one of them accepts public funding.[11]

We conclude that, while states that institute public financing programs may or may not do so with the intention of leveling the political playing field, the *ex post* impact of such programs can be anything—from a leveling effect, to the opposite, to nothing at all—depending on the private fundraising costs of the candidates. Without further information as to the relative likelihood of the cases listed in Proposition 2, the actual consequences of any given program for the balance of speech are impossible to assess.[12]

11. If society values an "unfettered interchange of ideas," the fact that the program decreased speech by both candidates could be considered a negative externality. We will examine the effect of public funding on the absolute amount of speech in Section 5.3.

12. One could argue that states that want to level the political playing field would only institute a public policy to this effect if the playing field is rather uneven to begin with, and Figure 2 suggests that, in such a case, public campaign financing often achieves at least a partial balancing of speech. But even if this is so, the public funding program

5.3. Does Public Campaign Funding Chill Political Speech?

Campaign finance regulations and the First Amendment are inherently at odds—the latter guarantees private entities to be free from government-imposed burdens on their speech, while the former imposes implicit costs on the most important form of speech, that is, political speech. Of interest in campaign finance cases, therefore, is the question of whether these costs are so high that they reduce the speech of some, or all, candidates in an election. In *Arizona Free Enterprise*, the Supreme Court concluded that the burden imposed by Arizona's matching program on non-participating candidates was sufficiently severe:

> Any increase in speech resulting from the Arizona law is of one kind and one kind only—that of publicly financed candidates. The burden imposed on privately financed candidates and independent expenditure groups reduces their speech. . . . Thus, even if the matching funds provision did result in more speech by publicly financed candidates and more speech in general, it would do so at the expense of impermissibly burdening (and thus reducing) the speech of privately financed candidates and independent expenditure groups.[13] (564 U.S. __ 2011, at 15.)

We now examine the validity of this claim within the context of our theoretical model. To do so, we derive its comparative statics and examine the effects of changes in the policy parameters T_0 and $T_{\max}$ on the equilibrium. The effect of matching funds on speech is then the derivative of equilibrium advertising with respect to $T_{\max}$.

Let us first consider the case where a change in the policy parameters does not affect the candidates' decisions to participate in the public program. That is, any adjustments in x_1 and x_2 are on the "intensive margin." The following result determines the direction of these adjustments for the three types of equilibrium we characterized previously:

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would achieve its goal regardless of whether it contains a matching funds provision or only a (large enough) lump-sum transfer.

13. The Supreme Court actually went further than that. While it believed that a decrease in privately funded speech was an undesirable consequence of Arizona's matching provision, it made clear that a burden on an activity remains a burden even when it does not decrease the activity (564 U.S. __ 2011, at 19). Thus, it is possible that the Court would have ruled the matching program unconstitutional even if it did not have speech-chilling effects.

Proposition 3 Suppose that either T_0 or $T_{\max}$ increases, without altering the candidates' equilibrium funding choices.

(a) In an all-private equilibrium, speech by each candidate remains unchanged.
(b) In an all-public equilibrium, speech by each candidate either remains constant (if only $T_{\max}$ increases but T_0 is constant) or increases strictly (if T_0 increases).
(c) In a private–public equilibrium, speech by each candidate either remains constant (if only T_0 increases but $T_{\max}$ is constant) or increases strictly (if $T_{\max}$ increases).

Part is (c) speaks directly to the scenario the Supreme Court alluded to in the quote above, namely a privately financed candidate running against a publicly financed candidate. In this case, a small increase in $T_{\max}$ increases the speech of both candidates, while a small decrease in T_0 does not decrease it. Because this applies, *inter alia*, to the case $T_{\max} = T_0$, we have the following result:

Corollary 2 Suppose the public funding program consists of a simple public option. Consider any equilibrium in which one candidate accepts this option and the other candidate rejects it. Then the introduction of a small additional amount of funds, awarded through matching, results in a strict increase of both candidates' speech. Conversely, awarding some of the existing amount of the public option through matching instead of lump-sum, does not decrease speech by either candidate.

The reason why not only the publicly funded candidate, but also the privately funded candidate, increases his speech when additional matching funds become available is the following. In a two-player Tullock contest model, as is ours, strategies are strategic complements for the player who spends the larger amount. In a private–public equilibrium, this is the player who is privately funded (see our discussion in Section 5.1). Adding matching funds to a public option therefore allows the publicly funded candidate to increase his speech, and because his opponent views his own spending and that of the publicly funded candidate as strategic complements, he increases his speech as well.

Next, consider the case where a change in the public funding program affects the candidates' financing choices; for example, a privately funded candidate decides to switch to the public program. The resulting changes in x_1 and x_2 are then "extensive margin" adjustments. In *Arizona Free Enterprise* the Supreme Court was also concerned with the potentially speech-reducing effects of such adjustments:

> If the matching funds provision achieves its professed goal and causes candidates to switch to public financing . . . there will be less speech: no spending above the initial state-set amount by formerly privately financed candidates, and no associated matching funds for anyone. Not only that, the level of speech will depend on the State's judgment of the desirable amount, an amount tethered to available (and often scarce) state resources. (564 U.S. __ 2011, at 15.)

To see how changes in public funding can induce switching from private to public financing in our model, differentiate the bounds K and $L(\cdot)$, given in Proposition 1, with respect to $T_{\max}$:

$$\frac{\partial K}{\partial T_{\max}} = -\frac{3/2 - \sqrt{2}}{(T_{\max})^2} < 0, \qquad \frac{\partial L(c)}{\partial T_{\max}} = -\frac{c^{3/4}}{4(T_{\max})^{5/4}} < 0.$$

Because candidate i prefers public over private funding when his cost c_i exceeds either K or $L(c_{-i})$ (depending on whether i's opponent is publicly or privately funded), a decrease in the thresholds K and $L(\cdot)$ implies that politicians are more inclined to accept public funding as $T_{\max}$ increases.

Figure 3 depicts the adjustments of the equilibrium regions identified in Proposition 1 when $T_{\max}$ increases and the K and $L(\cdot)$ curves shift inward: Some all-private equilibria become private–public, and some private–public equilibria become all-public. In both cases, an unambiguous drop in speech by each candidate results:

PROPOSITION 4 Suppose that the maximal state funding level $T_{\max}$ increases, and a candidate adjusts his funding choice as a result. Then the adjustment is a switch from private to public financing; moreover, at the moment the switch occurs both candidates decrease their speech.

Our model's predictions are, therefore, consistent with the Court's reasoning concerning the effects of extensive margin adjustments on speech, but not with that concerning the effects of intensive margin adjustments.

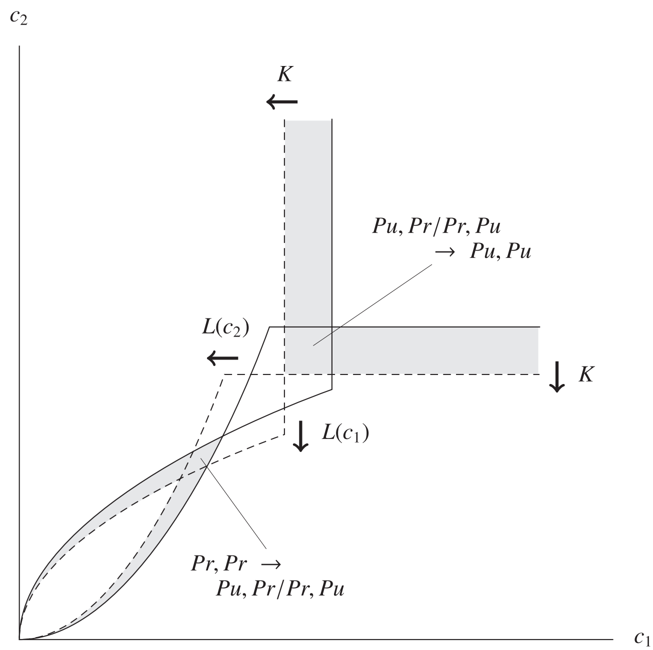

Figure 3. Funding Changes in Response to Increases in $T_{\max}$.

Note that Propositions 3 and 4 only characterize adjustments in political speech in response to *small* changes in our policy variables. These results do not allow us to evaluate the overall effect of a public funding system on speech. It is possible that a public funding system results in more speech than a private system of campaign finance, simply because a generous enough public option will be accepted by all candidates and allows all candidates a larger quantity of speech than they would have chosen otherwise. The Supreme Court cautioned that this scenario should not be the norm if state resources are scarce. Interestingly, however, even when state resources are scarce and some candidates choose to remain privately funded, public funding can increase total speech relative to private funding. What matters is the effect of public funding on the political playing field:

Proposition 5 Compared with the case of no public funding, a public campaign financing program increases total speech if it partially levels the playing field, and decreases total speech if it unlevels the playing field.

The condition for a partial leveling of the playing field—and thus for an increase in speech—is given in Proposition 2(c). For a given $T_{\max}$, the condition for part (c) of the result holds if one candidate's cost of speech is low enough in absolute terms, and the other candidate's cost is high relative to the first candidate's. Thus, whether public financing increases speech, compared with a world without such financing, is less a question of how scarce the state's resources are, but whether some candidates have a systematic and sufficiently strong fundraising advantage over their rivals. If this is so, then a purely private system of campaign funding does not generate a large amount of speech—the reason is the well-known fact that neither player in a Tullock contest exerts much effort if players have asymmetric costs. Public funding, by subsidizing the speech of the disadvantaged candidate, symmetrizes the contest and increases speech. Thus, it is precisely in those cases where public funding levels the playing field that it has the potential to increase speech.

Finally, we discuss an empirical test that has been proposed to examine if matching funds chill political speech: If they do, then privately financed candidates will cluster their spending just below the initial public grant T_0, as, at this threshold, the marginal benefit of a campaign dollar spent against a publicly financed candidate is neutralized by the state's matching transfers. The absence of clustering in actual campaign finance data from Maine and Arizona has been cited as evidence that these states' matching programs did not reduce privately financed speech in both the academic literature (see Gierzynski, 2011; Dowling et al., 2012) and in the arguments the state of Arizona brought before the Supreme Court in defense of its matching funds program.

In our model, it is indeed true that the marginal benefit of private spending against a publicly funded opponent is zero when private spending equals T_0, while the marginal cost is positive (see Equation (4), assuming $T_{\max} > T_0$). But, as we argued earlier, spending $< T_0$ from private funds against a publicly financed opponent is dominated by choosing the public option. The Supreme Court rejected the empirical "evidence" for the same reason:

> The State contends that if the matching funds provision truly burdened the speech of privately financed candidates and independent expenditure groups, spending on behalf of privately financed candidates would cluster just below the triggering level,

but no such phenomenon has been observed. . . . That should come as no surprise. The hypothesis presupposes a privately funded candidate who would spend his own money just up to the matching funds threshold, when he could have simply taken matching funds in the first place. (564 U.S. __ 2011, at 19.)

We point out, however, that the reasoning should be independent of whether the state operates a matching program or offers only a simple public option: By Proposition 1(c), the equilibrium speech by a privately financed candidate who runs against a publicly financed candidate is

$$\sqrt{T_{\max}/c_i} - T_{\max} \geqslant \sqrt{T_{\max}/K} - T_{\max}$$
$$= T_{\max} \cdot ((3/2 - \sqrt{2})^{-1/2} - 1) > T_{\max} \geqslant T_0.$$

Thus, while public options or matching funds may or may not reduce speech, the absence of private spending clustered at T_0 is not evidence for or against either of these possibilities.

6. Robustness and Extensions

In this section, we introduce several alternatives to our model assumptions and discuss if, and how, our model predictions would change under these alternatives. In Section 6.1, we revisit the equivalence of simple public options and matching mechanisms when private fundraising is risky, or when candidates' funding decisions are motivated by ideological, instead of monetary, considerations. In Section 6.2, we extend our model to allow candidates to condition their funding choice on their opponent's choice. In Section 6.3, we change our model so that candidates differ in the impact of their speech, instead of their fundraising costs.

6.1. Uncertain Fundraising Costs and Ideologically Motivated Candidates

We argued that awarding public campaign funds through a matching mechanism, or through a simple public option in the amount equal to the maximum funding level under the matching program, does not affect the candidates' participation decisions, their election probabilities, or their payoffs. The reason is that any candidate who spends less than the state maximum $T_{\max}$ from private funds would be better off if he accepted public financing. But if privately financed candidates always spend more than $T_{\max}$, a publicly funded opponent of a privately funded candidate will have resources $T_{\max}$ regardless of the mechanism through which $T_{\max}$ is paid. Thus, a privately funded candidate is not worse off if the state awards some of $T_{\max}$ through a matching program instead of awarding all of $T_{\max}$ in a lump-sum fashion.

Several counterarguments could be made to dismiss the above reasoning. First, a candidate may decline to participate in the public funding program in the expectation that he will raise private funds in excess of the state maximum $T_{\max}$. If this candidate's fundraising cost turns out higher than anticipated, he might adjust his fundraising and spending by an amount that depends on whether the public funding program is lump-sum or has a matching component.

Consider the following example. The public program is $(T_0, T_{\max}) = (1, 2)$ and the candidates have made funding decisions $(s_1, s_2) = (\text{Pr}, \text{Pu})$. These decisions would arise in equilibrium if $c_1 < K = 0.0428$ and $c_2 > L(c_1)$. Suppose that, after having declined public funding, candidate 2 learns that, unexpectedly, his fundraising cost has increased to $c_1 = 0.15$. Since $1/(4T_0) > c_1 > 1/(4T_{\max})$ now, using (5)–(6) we have

$$x_1 = 1.00, \quad x_2 = 1.00, \quad x_1 + x_2 = 2.00.$$

That is, candidate 1 spends exactly the state minimum, as any additional campaign dollar up to $T_{\max}$ would be neutralized by public funds disbursed to opponent (but spending more than $T_{\max}$ would be even worse for 1's payoff). If, instead, the public program were a simple option of the amount $T_{\max}$, then (5)–(6) imply that

$$x_1 = 1.65, \quad x_2 = 2.00, \quad x_1 + x_2 = 3.65.$$

Thus, each candidate's speech is higher in the second case than in the first.

What does this imply for social welfare and candidate welfare? Assuming political speech has a positive externality on society as a whole, this externality is larger under the lump-sum program in cases such as the one considered above. But so is the monetary cost of the program, and what type and size of public financing program society prefers, therefore, depends on the value society places on the quantity of political speech and on the public resources available to pay for political speech.[14] From the perspective of the privately funded candidate, on the other hand, the preference ordering is clear: The consequences of a cost shock or other fundraising failure are less severe in a matching program, compared with a simple option equal to the state maximum under matching. In the example above, candidate 1 wins with probability $1/2$ and spends 1 in the matching program, while he wins with probably $<1/2$ and spends more than 1 in the lump-sum program. Thus, while the quantity of political speech may depend on how public funds are awarded, our finding that privately funded candidates are not worse off under matching program $(T_0, T_{\max})$, compared with lump-sum program $(T_{\max}, T_{\max})$, continues to hold when fundraising costs are uncertain.

A second counterargument to our equivalence result is that some candidates may decline public financing for ideological reasons. The reasoning from the previous case applies here as well: Unless these candidates' costs of raising funds privately are low enough to forgo state funding in the first place, a matching program $(T_0, T_{\max})$ could result in less speech than a simple public option $(T_{\max}, T_{\max})$, and hence in lower social welfare. From the candidate's perspective, if the decision to decline state funding for ideological reasons lowers a candidate's chance of election and payoff, the effect is more severe under the lump-sum program than under matching. The same holds for political speech by independent expenditure groups: These groups cannot opt for public funding, which was a significant concern for the Supreme Court in *Arizona Free Enterprise* (564 U.S. __ 2011, at 3). But this is so regardless of whether public funding is provided in a lump-sum manner or through a matching program, and the burden imposed on independent expenditure groups by program $(T_{\max}, T_{\max})$ is the same, or larger, than the burden imposed by program $(T_0, T_{\max})$. Thus, while the quantity of political speech will depend on public financing program, our result that privately funded candidates (or independent expenditure groups) are not worse off under matching still holds.

14. The excerpts from *Arizona Free Enterprise* on Pages 25 and 27 of our paper suggest that the Supreme Court was perhaps relatively less concerned with the positive externalities from speech, and relatively more concerned with the state's ability to pay for speech.

6.2. Sequential Funding Choices

Some equilibria of our model may seem to be driven primarily by the assumption that the choice of campaign funding is a one-shot affair. For example, if candidates are symmetric, an all-private equilibrium will be Pareto-dominated by the outcome in which both candidates choose public funding. (In the all-private equilibrium the candidates win with equal probability and spend costly private funds; if both accepted public funding they would win with the same probabilities but at no cost). A move to this superior outcome requires a coordinated deviation from the all-private equilibrium. It appears, then, that if a candidate could commit to receive public funding, the other might follow suit, resulting in an overall payoff gain to both candidates.

Similarly, when our model gave rise to two private–public equilibria, allowing the candidates to move in sequence might eliminate one of these outcomes. For example, the candidate with the higher private fundraising cost might want to accept public funding early in a preemptive move. We now examine if the equilibrium outcomes of our model are affected if we allowed the candidates more flexibility in the timing of their decisions.

To do so, we consider the following extended version of our model. Decision-making takes place over $T+1$ stages ($T \geqslant 1$). At each stage $t = 1, \ldots, T$, candidates make simultaneous funding choices (s_1^t, s_2^t). We assume that a candidate can opt in to the public funding program at any time; but once a candidate has decided to receive public funding he cannot opt out at a later stage. Formally, assume that candidate $i = 1, 2$ starts out in "default funding mode" $s_i^0 = \text{Pr}$ and then chooses

$$s_i^t \in \begin{cases} \{\text{Pr}, \text{Pu}\} & \text{if } s_i^{t-1} = \text{Pr}, \\ \{\text{Pu}\} & \text{if } s_i^{t-1} = \text{Pu} \end{cases}$$

at stages $t = 1, \ldots, T$. At the end of stage t, each candidate observes if his opponent has opted to receive public funding, that is, candidates see (s_1^t, s_2^t) before setting s_1^{t+1} and s_2^{t+1}. Finally, at stage $T+1$ the candidates compete in the election contest by setting (x_1, x_2), taking (s_1^T, s_2^T) as given.

Because a candidate cannot withdraw from the public program once he has elected to participate, this extension of our model permits us to study commitment and preemption effects of the type discussed above. However, it does not require us to arbitrarily declare one candidate a "first mover" and the other a "second mover." Instead, the sorting of players into first, second, or simultaneous movers is allowed to happen endogenously.[15]

Call the game with T funding rounds, followed by one election contest, Γ^T. We look for the pure strategy, subgame perfect equilibria of Γ^T. Note that our original model is the game Γ^1, and, mirroring our previous terminology, we identify an equilibrium by the funding modes with which the candidates head into the election contest. In particular, we say (s_1^*, s_2^*) is an *equilibrium funding pair* (EFP) of Γ^T if Γ^T has a pure strategy, subgame perfect equilibrium in which $(s_1^T, s_2^T) = (s_1^*, s_2^*)$. Given the EFP, campaign spending and election probabilities at stage $T + 1$ are then determined as they were at stage 2 of our original model (see Section 4.1). We have the following result:

Proposition 6 (s_1^*, s_2^*) is an EFP of Γ^T $(T > 1)$ if and only if (s_1^*, s_2^*) is an EFP of Γ^1.

Thus, our original model did not "miss" any outcomes that might arise in equilibrium of the longer game Γ^T, and neither did it create any equilibria that would not survive if there were multiple rounds of funding decisions. In particular, all equilibria of our original model are robust to the possibility that a candidate may want to delay the decision to accept public funding until his opponent has done so, and to the possibility that a candidate may want to commit to public funding early in order to induce their opponent to accept public funding.

6.3. Asymmetric Campaign Strengths

In our model, candidates differed in terms of their private fundraising costs. This is not the only kind of asymmetry that exists between political candidates. In particular, candidates could differ in terms of the impact of their speech on election outcomes, instead of the cost of their speech. In a Tullock contest, one can model such an asymmetry by replacing the original

15. See Fu (2006) for a contest model where a similar structure is used to endogenize the timing of the players' efforts.

contest success function (1) with

$$P_i = g(x_i, x_{-i}) \equiv \begin{cases} \dfrac{\alpha_i x_i}{\alpha_i x_i + \alpha_{-i} x_{-i}} & \text{if } x_1 + x_2 > 0, \\ \dfrac{\alpha_i}{\alpha_i + \alpha_{-i}} & \text{if } x_1 + x_2 = 0. \end{cases} \tag{10}$$

The variables x_1 and x_2 can be thought of as the candidates' "nominal speech" and $\alpha_1 x_1$ and $\alpha_2 x_2$ as their "effective speech," where $\alpha_1 > 0$ and $\alpha_2 > 0$ measure the *impact* of each candidate's nominal speech.[16]

In our original model, we set $\alpha_1 = \alpha_2 = 1$ for the following reason. An important motivation for states to offer public campaign funds is a concern that candidates who lack access to wealthy donors may otherwise be unable to fund competitive campaigns. States generally do not offer public financing with the goal of assisting candidates whose speech has (for whatever reason) little impact on election results. It is therefore natural to begin studying the effects of public funding programs in elections where candidates differ in their fundraising costs. Nevertheless, it is also interesting to ask how a public funding program would affect elections if candidates differed in the impact of their speech on election probabilities. For simplicity, let us assume that fundraising costs are the same in this case.

If all funds are privately raised, a Tullock contest with asymmetric impacts and symmetric fundraising costs is isomorphic to one with symmetric impact and asymmetric costs. To see this, set $x_i' \equiv c_i x_i$ and $\alpha_i \equiv 1/c_i$ and observe that $f(x_i, x_{-i}) - c_i x_i = g(x_i', x_{-i}') - x_i'$. That is, after rescaling the candidates' strategies the payoff function of an asymmetric-cost contest is the same as that of an asymmetric-impact contest with impacts $(1/c_1, 1/c_2)$ and unit costs.[17] This implies that in an all-private equilibrium

16. There are several possible interpretations of the impact coefficients α_i. First, one candidate's impact could be small, relative to his opponent's, because he utilizes a less efficient advertising technology. Secondly, the candidate with the smaller α_i could be campaigning on a fringe platform that affects only a small subset of the electorate. Thirdly, α_i could be small because candidate i's platform addresses a complex policy challenge and requires relatively more effort to be conveyed to voters. Fourthly, α_i could be small because candidate i is unattractive to voters for some extraneous reason, such as appearance or ancestry.

17. Both specifications are also equivalent to one with symmetric campaign strengths and symmetric fundraising costs, but asymmetric prizes.

of the asymmetric-impact model, the two candidates win with probabilities $P_i = \alpha_i/(\alpha_1 + \alpha_2)$. In the presence of a public financing program, this equivalence no longer holds. For example, in the all-public equilibrium of the asymmetric-cost model, both candidates win with probability $1/2$, while in the same type of equilibrium of the asymmetric-impact model they win with probability $\alpha_1/(\alpha_1 + \alpha_2)$ and $\alpha_2/(\alpha_1 + \alpha_2)$, respectively. Because the public option only equalizes nominal speech, but not effective speech, it fully preserves the existing asymmetry between the candidates. In particular, if both candidates accept public funding, the political playing field would remain unaltered.

While a full analysis of the equilibria in the asymmetric-impact model is beyond the scope of this paper, one important insight from our original model is unaffected by the nature of asymmetry between candidates: The candidates' election probabilities and funding choices depend on the public financing program only through its maximum state funding level, $T_{\max}$. The reason is a dominance argument similar to the one made in Section 5.1 (subject to the same potential caveats discussed in Section 6.1): A matching program $(T_0, T_{\max})$ generates different outcomes than the simple public option $(T_{\max}, T_{\max})$ only if one candidate accepts public funding while the other candidate rejects it and spends below $T_{\max}$. In this case, the privately funded candidate (i, say) wins with a probability of not more than $\alpha_i/(\alpha_1 + \alpha_2)$, and pays a positive cost. If i deviated at stage 1 and accepted public funding, he would win with probability $\alpha_i/(\alpha_1 + \alpha_2)$ and pay zero. It follows that both $(T_0, T_{\max})$ and $(T_{\max}, T_{\max})$ result in the same election outcomes; however, as was the case before, the simple option will be costlier to the state if both candidates accept it.

7. Conclusion

We developed a model of costly political speech in election contests to examine the effects of different public campaign financing programs on election probabilities and the quantity of political speech by candidates. We used the model to evaluate the arguments made by the U.S. Supreme Court when it invalidated the allocation of public funds through matching mechanisms based on its "leveling the playing field" doctrine. Within the context of our model we found several of these arguments to be incorrect or inconsistent. While we do not take a stance with regard to the "leveling" doctrine itself, our results call into question the Court's application of it to public campaign financing.

We derived our results in a simple model based on the Tullock contest. However, the intuition behind many of our results appears to be robust to alternative specifications of the contest success function. For example, the observation that a candidate should accept public funding unless he is willing to spend in excess of the maximum public funding level is a straightforward, dominance-based argument, which holds regardless of the functional form used to describe the relationship between campaign spending and election probabilities. We used this argument to establish the equivalence between simple public options and matching programs, and to challenge the interpretation of existing empirical studies of matching programs. Likewise, the fact that a more symmetric contest results in more effort is a general result that has been shown to hold in a variety of alternative specifications (see Konrad, 2009 for a survey). In our model, this result implied that the incentives to engage in political speech under a public financing program are strengthened if the program achieves some degree of leveling of the playing field.

Our analysis could be extended in a number of directions. First, one could examine the case of more than two candidates. In a matching program, for example, the grants made to publicly financed candidates would then depend on the highest spending of any of the privately financed candidates (see the matching formula previously used by Arizona described in Footnote 7). Secondly, one could more fully investigate the case where candidates differ in the impact of their campaign messages. While we demonstrated in Section 6.3 that election probabilities in an all-private and all-public equilibrium are the same in this case, we did not analyze the candidates' decisions to accept public funding in the first place. Thirdly, the candidates' fundraising costs (or other payoff-relevant characteristics) could be endogenized, by adding a stage at which candidates decide whether to enter the contest. For concreteness, suppose that potential candidates belong to a pool of politicians with varying fundraising costs, and a candidate from this pool would enter the contest against a give opponent if his expected payoff in equilibrium of our current model exceeds some reservation utility (which could be correlated with the candidate's fundraising cost). The Nash equilibria of this entry game could then provide some information about which of the cases in Proposition 2, for example, are more likely to arise than others. Investigating these extensions is beyond the scope of this paper, but each could be a fruitful direction for future research.

Appendix

Proof of Proposition 1

To prove part (a), suppose that candidate $-i$ chooses public funding. Then candidate i obtains

$$v_i^*(\mathrm{Pu}, \mathrm{Pu}) = \tfrac{1}{2} \tag{A1}$$

if i chooses public funding, and

$$v_i^*(s_i = \mathrm{Pr}, s_{-i} = \mathrm{Pu}) = \begin{cases} 0 & \text{if } c_i > \dfrac{1}{T_0}, \\ (1 - \sqrt{c_i T_0})^2 & \text{if } \dfrac{1}{T_0} \geqslant c_i > \dfrac{1}{4T_0}, \\ \dfrac{1}{2} - c_i T_0 & \text{if } \dfrac{1}{4T_0} \geqslant c_i > \dfrac{1}{4T_{\max}}, \\ (1 - \sqrt{c_i T_{\max}})^2 & \text{if } c_i \leqslant \dfrac{1}{4T_{\max}} \end{cases} \tag{A2}$$

if i chooses private funding. The first three terms in (A2) are strictly less than (A1), and the fourth term is weakly greater than (A1) if and only if

$$c_i \leqslant K = \frac{3/2 - \sqrt{2}}{T_{\max}} < \frac{1}{4T_{\max}}.$$

Part (a) of the result now follows immediately.

To prove part (b), suppose that candidate $-i$ chooses private funding. Then candidate i obtains

$$v_i^*(s_i = \text{Pu}, s_{-i} = \text{Pr}) = \begin{cases} 1 & \text{if } c_{-i} > \dfrac{1}{T_0}, \\ \sqrt{c_{-i} T_0} & \text{if } \dfrac{1}{T_0} \geqslant c_{-i} > \dfrac{1}{4T_0}, \\ \dfrac{1}{2} & \text{if } \dfrac{1}{4T_0} \geqslant c_{-i} > \dfrac{1}{4T_{\max}}, \\ \sqrt{c_{-i} T_{\max}} & \text{if } c_{-i} \leqslant \dfrac{1}{4T_{\max}} \end{cases} \tag{A3}$$

if i chooses public funding, and

$$v_i^*(\text{Pr}, \text{Pr}) = \frac{(c_{-i})^2}{(c_i + c_{-i})^2} \tag{A4}$$

if i chooses private funding. Consider the fourth term in (A3), which applies if $c_{-i} \leqslant 1/(4T_{\max})$. This term is less than or equal to (A4) if and only if

$$c_i \leqslant L(c_{-i}) = c_{-i} \cdot ((c_{-i} T_{\max})^{-1/4} - 1).$$

Let $\bar{c} \equiv 1/(16T_{\max}) < 1/(4T_{\max})$. We will show two claims: (i) $c_i \leqslant \bar{c}\ \forall i$ is necessary for an all-private equilibrium; (ii) $c_i \leqslant L(c_{-i})\ \forall i$ implies $c_i \leqslant \bar{c}$ $\forall i$. It then follows that $c_i \leqslant L(c_{-i})$ for $i = 1, 2$ is necessary and sufficient for an all-private equilibrium.

To prove claims (i) and (ii), observe that

$$L(c) \gtreqless c \ \text{ if } c \lesseqgtr \bar{c} \quad \text{and} \quad L(c) \leqslant \bar{c} \ \text{ if } c \leqslant \bar{c}. \tag{A5}$$

For claim (i), note that, if $c_i > \bar{c}\ \forall i$, then (A3) implies that $v_i^*(s_i = \text{Pu}, s_{-i} = \text{Pr}) > \frac{1}{4}\ \forall i$. On the other hand, (A4) implies that $v_i^*(\text{Pr}, \text{Pr}) = (c_{-i})^2/[(c_i + c_{-i})^2] \leqslant \frac{1}{4}$ for some i (regardless of the candidates' costs). It follows that, if both candidates have costs above $\bar{c}$ and choose private funding, at least one candidate wants to deviate to public funding. Therefore, in an all-private equilibrium at least one of c_1 and c_2 must be weakly below $\bar{c}$. Without loss of generality, assume $c_1 \leqslant \bar{c}$. By (A5), this implies that $L(c_1) \leqslant \bar{c}$. Candidate 2 prefers private to public funding if and only if $c_2 \leqslant L(c_1)$; since $L(c_1) \leqslant \bar{c}$ we have $c_2 \leqslant \bar{c}$ and claim (i) follows. For claim (ii), assume $c_1 \leqslant L(c_2)$ and $c_2 \leqslant L(c_1)$. Suppose, contrary to the claim, that

$c_1 > \overline{c}$. By (A5), this implies that $L(c_1) < c_1$. Also, since $c_1 \leqslant L(c_2)$, we have $L(c_2) > \overline{c}$. By (A5), this implies $c_2 > \overline{c}$, and thus $L(c_2) < c_2$. Combining these inequalities, we obtain $c_2 \leqslant L(c_1) < c_1 \leqslant L(c_2) < c_2$, a contradiction. The case against $c_2 > \overline{c}$ is similar.

Finally, we prove part (c). From the proof of part (a), we know that $s_i = \mathrm{Pr}$ is a best response to $s_{-i} = \mathrm{Pu}$ if and only if $c_i \leqslant K$. From the proof of part (b), we know that $s_{-i} = \mathrm{Pu}$ is a best response to $s_i = \mathrm{Pr}$ if and only if $c_{-i} \geqslant L(c_i)$, provided $c_i < 1/(4T_{\max})$. Since $K < 1/(4T_{\max})$, this is satisfied when $c_i \leqslant K$. The result follows.

Proof of Proposition 2

Part (b) is the same as the condition for an all-private equilibrium in Proposition 1(b), assuming $c_i < c_{-i}$. If the public financing program results in an all-private equilibrium, then it obviously leaves the playing field unchanged. Similarly, part (d) is the same as the condition for an all-public equilibrium in Proposition 1(a), again assuming $c_i < c_{-i}$. If the public financing program results in an all-public equilibrium, it levels the playing field fully (both candidates spend T_0 and both win with probability $\frac{1}{2}$)

Now consider a private–public equilibrium in which the disadvantaged candidate $-i$ chooses public funding. By Proposition 1(c), this equilibrium exists if

$$c_i \leqslant K = \frac{3/2 - \sqrt{2}}{T_{\max}} \Leftrightarrow T_{\max} \leqslant \frac{3/2 - \sqrt{2}}{c_i} \tag{A6}$$

$$\text{and} \quad c_{-i} \geqslant L(c_i) = c_i(c_i T_{\max})^{-1/4} - 1) \Leftrightarrow T_{\max} \geqslant \left(\frac{c_i}{c_1 + c_2}\right)^2 \frac{c_i}{(c_1 + c_2)^2}. \tag{A7}$$

Since the privately financed candidate spends more than the publicly funded candidate in a private–public equilibrium, $-i$ is less likely to win than i. This implies that the funding system either partially levels the playing field or unlevels it. Note that in an all-private equilibrium we have

$$\frac{x_i}{x_{-i}} = \frac{c_{-i}}{c_i},$$

and in the private–public equilibrium, where i is privately funded we have

$$\frac{x_i}{x_{-i}} = \frac{\sqrt{T_{\max}/c_i} - T_{\max}}{T_{\max}} = \frac{1}{\sqrt{c_i T_{\max}}} - 1.$$

For a partial leveling of the playing field, therefore, we need

$$\frac{1}{\sqrt{c_i T_{\max}}} - 1 < \frac{c_{-i}}{c_i} \Leftrightarrow T_{\max} > \frac{c_i}{(c_1 + c_2)^2}. \tag{A8}$$

Note that (A8) implies (A7); thus, (A6) and (A8) together constitute a condition for a private–public equilibrium with partial leveling. This condition can be written

$$\frac{c_i}{(c_1 + c_2)^2} < T_{\max} \leqslant \frac{3/2 - \sqrt{2}}{c_i},$$

which is the condition stated in part (c). Similarly, for an unleveling of the playing field the strict inequality in (A8) is reversed:

$$\frac{1}{\sqrt{c_i T_{\max}}} - 1 > \frac{c_{-i}}{c_i} \Leftrightarrow T_{\max} < \frac{c_i}{(c_1 + c_2)^2}. \tag{A9}$$

In this case, (A6), (A7), and (A8) together constitute a condition for a private–public equilibrium with unleveling. This condition can be written as

$$\left(\frac{c_i}{c_1 + c_2}\right)^2 \frac{c_i}{(c_1 + c_2)^2} \leqslant T_{\max} < \min\left\{\frac{c_i}{(c_1 + c_2)^2}, \frac{3/2 - \sqrt{2}}{c_i}\right\},$$

which is the condition stated in part (a).

Finally, consider a private–public equilibrium in which the advantaged candidate i chooses public funding. By Proposition 1(c), this equilibrium exists if

$$c_{-i} \leqslant K = \frac{3/2 - \sqrt{2}}{T_{\max}} \Leftrightarrow T_{\max} \leqslant \frac{3/2 - \sqrt{2}}{c_{-i}} \tag{A10}$$

$$\text{and} \quad c_i \geqslant L(c_{-i}) = c_{-i}((c_{-i} T_{\max})^{-1/4} - 1) \Leftrightarrow T_{\max} \geqslant \left(\frac{c_{-i}}{c_1 + c_2}\right)^2 \frac{c_{-i}}{(c_1 + c_2)^2}. \tag{A11}$$

Since the privately financed candidate spends more than the publicly funded candidate in a private–public equilibrium, $-i$ is more likely to win than i, so that the funding system reverses the playing field. Therefore, (A10) and (A11) together constitute a condition for a private–public equilibrium with reversal of the playing field. This condition can be written as

$$\left(\frac{c_{-i}}{c_1+c_2}\right)^2 \frac{c_{-i}}{(c_1+c_2)^2} \leqslant T_{\max} \leqslant \frac{3/2-\sqrt{2}}{c_{-i}},$$

which is the condition stated in part (b).

Because we have now considered all possible pure strategy equilibria, the conditions (a)–(e) are also necessary.

Proof of Proposition 3

By Proposition 1(a), candidate i's speech in an all-private equilibrium is $x_i = c_{-i}/(c_1+c_2)^2$, which is independent of T_0 and $T_{\max}$. Similarly, by Proposition 1(b), candidate i's speech in an all-public equilibrium is $x_i = T$, which is increasing in T_0 and independent of $T_{\max}$. This establishes parts (a) and (b) of the result. To show part (c), consider a private–public equilibrium. Let i be the privately funded candidate and let $-i$ be the publicly funded candidate. By Proposition 1(c), the amount of private speech is $x_i = \sqrt{T_{\max}/c_i} - T_{\max}$ and the amount of publicly funded speech is $x_{-i} = T_{\max}$. x_{-i} is strictly increasing in $T_{\max}$. x_i is increasing in $T_{\max}$ if and only if

$$\frac{\partial}{\partial T_{\max}}\left[\sqrt{T_{\max}/c_i} - T_{\max}\right] = \sqrt{1/(4c_i T_{\max})} - 1 > 0 \Leftrightarrow c_i < \frac{1}{4T_{\max}}.$$

Note that $c_i \leqslant K$ in a private–public equilibrium. Since

$$K = \frac{3/2-\sqrt{2}}{T_{\max}} \approx 0.09\frac{1}{T_{\max}} < \frac{1}{4T_{\max}},$$

the result follows.

Proof of Proposition 4

To show part (a), consider a private–public equilibrium. Assume that candidate i is privately funded and candidate $-i$ is publicly funded. Proposition 1(c) implies $c_i \leqslant K$ and $c_{-i} \leqslant L(c_i)$; furthermore, $x_i = \sqrt{(T_{\max})/c_i} - T_{\max}$ and $x_{-i} = T_{\max}$. Now suppose that $c_i = K$. Then a small increase in $T_{\max}$ turns the private–public equilibrium into an all-public equilibrium (by Proposition 1(a)). In the new all-public equilibrium, spending is

$\tilde{x}_i = \tilde{x}_{-i} = T_0$. Thus, candidate $-i$ weakly decreases his spending when the switch occurs. Furthermore, since $c_i = K = (3/2 - \sqrt{2})/(T_{\max})$ we can express candidate i's spending in the original private–public equilibrium as

$$x_i = \sqrt{(T_{\max})/c_i} - T_{\max} = T_{\max}\left(\sqrt{1/\left(\tfrac{3}{2} - \sqrt{2}\right)} - 1\right).$$

This is larger than T_0 if and only if $3/2 - \sqrt{2} < T_{\max}^2/[(T_0 + T_{\max})^2]$. Since $T_{\max} \geqslant T_0$, we have $T_{\max}^2/[(T_0 + T_{\max})^2] \geqslant 1/4 > 3/2 - \sqrt{2}$ and therefore $x_i > \tilde{x}_i$. Thus, when the funding switch occurs, candidate i strictly decreases his spending as well.

To show part (b), consider an all-private equilibrium. Proposition 1(b) implies $c_i \leqslant L(c_{-i})$ and $c_{-i} \leqslant L(c_i)$; furthermore, $x_i = c_{-i}/[(c_i + c_{-i})]^2$ and $x_{-i} = c_i/[(c_i + c_{-i})]^2$. Now suppose that $c_{-i} = L(c_i)$. Then a small increase in $T_{\max}$ will turn the all-private equilibrium into a private–public equilibrium where candidate i remains privately funded, and candidate $-i$ has switched to public funding (by Proposition 1(c)). In the new private–public equilibrium, total spending equals $\tilde{x}_i + \tilde{x}_{-i} = \sqrt{T_{\max}/c_i}$. Using $c_{-i} = L(c_i)$, we can express candidate $-i$'s spending in the original all-private equilibrium as

$$x_{-i} = c_i \cdot (c_i[(c_i T_{\max})^{-1/4} - 1] + c_i)^{-2} = \sqrt{(T_{\max})/c_i} = \tilde{x}_i + \tilde{x}_{-i}.$$

That is, candidate $-i$ alone spends as much in the all-private equilibrium as the two candidates spend together in the public–private equilibrium. It follows that, at the moment the funding switch occurs, candidate $-i$ decreases his speech strictly. Lastly, let us consider candidate i's spending in the all-private equilibrium, which is $x_i = c_{-i}/[(c_i + c_{-i})]^2$. Recall from the proof of Proposition 1(b) that in an all-private equilibrium $c_i, c_{-i} \leqslant \bar{c}$, and thus $L(c_i) \geqslant c_i$. Since $c_{-i} = L(c_i)$, this implies $c_{-i} \geqslant c_i$, and thus $x_i \geqslant x_{-i} > \tilde{x}_i + \tilde{x}_{-i}$. It follows that, at the moment the funding switch occurs, candidate i also decreases his speech strictly.

Proof of Proposition 5

By Proposition 1, total speech in a private–public equilibrium with candidate i being privately funded and candidate $-i$ being publicly funded equals

$$X^{\mathrm{Pr,Pu}} = \underbrace{\sqrt{T_{\max}/c_i} - T_{\max}}_{x_i} + \underbrace{T_{\max}}_{x_{-i}} = \sqrt{T_{\max}/c_i}.$$

Total speech in an all-private equilibrium, which would necessarily obtain if there was no public financing available to candidates, equals

$$X^{\mathrm{Pr,Pr}} = \underbrace{\frac{c_{-i}}{(c_1+c_2)^2}}_{x_i} + \underbrace{\frac{c_i}{(c_1+c_2)^2}}_{x_{-i}} = \frac{1}{c_1+c_2}.$$

Speech in the private–public equilibrium exceeds that in the all-private equilibrium if

$$\sqrt{T_{\max}/c_i} > \frac{1}{c_1+c_2} \Leftrightarrow T_{\max} > \frac{c_i}{(c_1+c_2)^2}. \tag{A12}$$

A private–public equilibrium arises if conditions (A6)–(A7) hold (see proof of Proposition 2). Note that (A12) implies (A7); thus, (A6) and (A12) together constitute a condition for the public funding system $(T_0, T_{\max})$ to have a private–public equilibrium in which total speech is higher than in the absence of public funding, and this is precisely the same condition as that for partial leveling in Proposition 2(c).

Similarly, speech in the private–public equilibrium is less than speech in the all-private equilibrium if the inequality in (A12) is reversed:

$$\sqrt{T_{\max}/c_i} < \frac{1}{c_1+c_2} \Leftrightarrow T_{\max} < \frac{c_i}{(c_1+c_2)^2}. \tag{A13}$$

(A13), (A6), and (A7), constitutes a condition for the public funding system $(T_0, T_{\max})$ to have a private–public equilibrium where total speech is less than in the absence of public funding. This is precisely the same condition as that for unleveling in Proposition 2(a).

Proof of Proposition 6

We express Γ^T recursively as follows. Denote by G_i a game in which candidate $-i$ is publicly funded and candidate i chooses between public and private funding, followed by an election contest. Denote by G_0 an election in which both candidates are publicly funded and neither candidate makes a decision. Call two games *strategically equivalent* if they have the same reduced normal form, and observe that

$$\text{Stage-2 subgame of } \Gamma^T \text{ is strategically equivalent to}$$
$$\begin{cases} \Gamma^{T-1} & \text{if } (s_1^1, s_2^1) = (\text{Pr}, \text{Pr}), \\ G_0 & \text{if } (s_1^1, s_2^1) = (\text{Pu}, \text{Pu}), \\ G_1 & \text{if } (s_1^1, s_2^1) = (\text{Pr}, \text{Pu}), \\ G_2 & \text{if } (s_1^1, s_2^1) = (\text{Pu}, \text{Pr}). \end{cases}$$

Note that (Pu, Pu) is the unique EFP of G_0; (Pr, Pu) and (Pu, Pu) are the only possible EFPs of G_1, and (Pu, Pr) and (Pu, Pu) are the only possible EFPs of G_2. Further, note that the continuation payoffs at stage $T+1$ of Γ^T are the same as the continuation payoffs at stage 2 of Γ^1, and given by $v_i^*(s_1^T, s_2^T)$ (see Section 4.1).

We now proceed in two parts. In Part 1, we prove that an EFP or Γ^1 is an EFP of Γ^T $(T > 1)$, and in Part 2 we prove the converse.

Part 1. Suppose (s_1^*, s_2^*) is an EFP of Γ^1 and Γ^{T-1}, for $T = 2, 3, \ldots$. We show that (s_1^*, s_2^*) is an EFP of Γ^T; the result then follows by induction on T. Consider the following cases:

1. $(s_1^*, s_2^*) = (\text{Pr}, \text{Pr})$. By Proposition 1, $c_1 \leqslant L(c_2)$ and $c_2 \leqslant L(c_1)$. In the proof of Proposition 1, we showed that $c_i \leqslant L(c_{-i})$ $\forall i$ implies $c_i \leqslant 1/(16T_{\max}) < K$ $\forall i$. Therefore, we have $c_1, c_2 < K$, which means that (Pr, Pu) is an EFP of G_1 and (Pu, Pr) is an EFP of G_2. It follows that Γ^T has an equilibrium in which $(s_1^T, s_2^T) = (s_1^1, s_2^1)$. Suppose $(s_1^1, s_2^1) = (\text{Pr}, \text{Pr})$, so that $(s_1^T, s_2^T) = (\text{Pr}, \text{Pr})$. If candidate 1 deviates and chooses $s_1^1 = \text{Pu}$, he changes the final funding pair to $(s_1^T, s_2^T) = (\text{Pu}, \text{Pr})$, and because (Pr, Pr) is an EFP of Γ^1 he does not strictly prefer do so. The same argument applies to candidate 2. Therefore, Γ^T has an equilibrium in which $(s_1^1, s_2^1) = (\text{Pr}, \text{Pr})$, and hence $(s_1^T, s_2^T) = (\text{Pr}, \text{Pr})$.
2. $(s_1^*, s_2^*) = (\text{Pu}, \text{Pu})$. By Proposition 1, $c_1, c_2 \geqslant K$, which means (Pu, Pu) is an EFP of G_1 and G_2. Suppose $(s_1^1, s_2^1) = (\text{Pu}, \text{Pu})$, so that $(s_1^T, s_2^T) = (\text{Pu}, \text{Pu})$. If candidate 1 deviates and chooses $s_1^1 = \text{Pr}$, he does not change (s_1^T, s_2^T); the same applies to candidate 2. Therefore, Γ^T has an equilibrium in which $(s_1^1, s_2^1) = (\text{Pu}, \text{Pu})$, and hence $(s_1^T, s_2^T) = (\text{Pu}, \text{Pu})$.

3. $(s_1^*, s_2^*) = (\text{Pr}, \text{Pu})$. By Proposition 1, $c_1 \leqslant L$ and $c_2 \geqslant L(c_1)$, so (Pr, Pu) is an EFP of G_1. It follows that Γ^T has an equilibrium in which

$$(s_1^T, s_2^T) = \begin{cases} (\text{Pr}, \text{Pu}) & \text{if } (s_1^1, s_2^1) = (\text{Pr}, \text{Pr}) \text{ or } (\text{Pr}, \text{Pu}), \\ (\text{Pu}, \text{Pu}) & \text{if } (s_1^1, s_2^1) = (\text{Pu}, \text{Pu}), \\ (\text{Pu}, \text{Pr}) \text{ or } (\text{Pu}, \text{Pu}) & \text{if } (s_1^1, s_2^1) = (\text{Pu}, \text{Pr}). \end{cases}$$

Suppose $(s_1^1, s_2^1) = (\text{Pr}, \text{Pu})$, so that $(s_1^T, s_2^T) = (\text{Pr}, \text{Pu})$. If candidate 1 deviates and chooses $s_1^1 = \text{Pu}$, he changes the final funding pair to $(s_1^T, s_2^T) = (\text{Pu}, \text{Pu})$, and because (Pr, Pu) is an EFP of Γ^1 he does not strictly prefer do so. If candidate 2 deviates and chooses $s_2^1 = \text{Pr}$, he does not change (s_1^T, s_2^T). Therefore, Γ^T has an equilibrium in which $(s_1^1, s_2^1) = (\text{Pr}, \text{Pu})$, and hence $(s_1^T, s_2^T) = (\text{Pr}, \text{Pu})$. (The case $(s_1^*, s_2^*) = (\text{Pu}, \text{Pr})$ is analogous.)

Part 2. Suppose (s_1, s_2) is not an EFP of Γ^1 and Γ^{T-1}, for $T = 2, 3, \ldots$. We show that (s_1, s_2) is not an EFP of Γ^T; the result then follows by induction on T. Consider the following cases:

1. $(s_1, s_2) = (\text{Pr}, \text{Pr})$. In this case, none of the stage-2 subgames of Γ^T admits (Pr, Pr) as an EFP. Thus, (Pr, Pr) is not an EFP of Γ^T.
2. $(s_1, s_2) = (\text{Pu}, \text{Pu})$. This means $v_1^*(\text{Pr}, \text{Pu}) > v_i^*(\text{Pu}, \text{Pu})$ or $v_2^*(\text{Pu}, \text{Pr}) > v_2^*(\text{Pu}, \text{Pu})$ or both. Without loss of generality, assume the first case. Suppose Γ^T has an equilibrium in which $(s_1^T, s_2^T) = (\text{Pu}, \text{Pu})$. Since the only stage-2 subgame that admits (Pu, Pu) as an EFP is G_0, we have $s_1^1 = s_2^1 = \text{Pu}$ in this equilibrium. By deviating to $s_1^1 = \text{Pr}$, candidate 1 can change the final funding pair to (Pr, Pu), which he strictly prefers. Thus, (Pu, Pu) is not an EFP of Γ^T.
3. $(s_1, s_2) = (\text{Pr}, \text{Pu})$. This means $v_1^*(\text{Pu}, \text{Pu}) = 1/2 > v_1^*(\text{Pr}, \text{Pu})$ or $v_2^*(\text{Pr}, \text{Pr}) > v_2^*(\text{Pr}, \text{Pu})$ or both. Suppose Γ^T has an equilibrium in which $(s_1^T, s_2^T) = (\text{Pr}, \text{Pu})$. Since the only stage-2 subgame of Γ^T that admits (Pr, Pu) as an EFP is G_1, it must be that $s_1^1 = \text{Pr}$ and $s_2^1 = \text{Pu}$ in this equilibrium. Since candidate 1 can induce subgame G_0 by deviating $s_1^1 = \text{Pu}$, it must be that $v_1^*(\text{Pr}, \text{Pu}) \geqslant 1/2 = v_1^*(\text{Pu}, \text{Pu})$, and hence $v_2^*(\text{Pr}, \text{Pr}) > v_2^*(\text{Pr}, \text{Pu})$. Note that candidate 2 can induce subgame Γ^{T-1} by deviating $s_2^1 = \text{Pr}$; thus, the final funding pair in Γ^{T-1} cannot be (Pr, Pr). It can also not be (Pr, Pu), as this is not an

EFP of Γ^{T-1}. This means that candidate 1 accepts public funding in Γ^{T-1}. But then candidate 2 can induce final funding pair (Pu, Pu) in Γ^{T}, by setting $s_2^1 = \text{Pr}$ to enter subgame Γ^{T-1} and then accepting public funding in the final funding stage of Γ^{T-1}. Since (Pr, Pu) is an EFP of Γ^{T}, it follows that $v_2^*(\text{Pr}, \text{Pu}) \geqslant v_2^*(\text{Pu}, \text{Pu}) \geqslant 1/2$. Thus, $v_1^*(\text{Pr}, \text{Pu}) + v_2^*(\text{Pr}, \text{Pu}) \geqslant 1$, an impossibility (see (7)–(8)), and we conclude that (Pr, Pu) is not an EFP of Γ^{T}. (The case $(s_1, s_2) =$ (Pu, Pr) is analogous.)

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