Counterintuitive Signs in Reduced Form Price Regressions

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I. Introduction

Reduced form price regressions are widely used in price-fixing cases for quantifying damages and in merger investigations for estimating likely price effects. In some cases, the estimated coefficients of one or more independent variables may have signs inconsistent with economic theory. For example, basic economic theory shows that an increase in demand, all else equal, leads to an increase in price. Similarly, an increase in cost, all else equal, leads to an increase in price. In a major price-fixing case, plaintiffs’ expert’s reduced-form regression model showed that an increase in the values of variables measuring demand led to lower prices, and an increase in the value of a cost index also led to lower prices. The defense’s expert concluded, on this basis, that the regression model was fundamentally flawed. Plaintiffs’ expert responded by arguing that the field of econometrics recognizes that the interaction of supply and demand effects in a reduced-form regression model can yield such counterintuitive signs even though the underlying supply and demand effects conform to economic theory.\textsuperscript{4} We discuss how to interpret such counterintuitive signs and whether they cast doubt on the reduced-form model.

II. Measuring Antitrust Damages and Estimating Merger Effects on Prices

In assessing damages in price-fixing cases, a central economic question is the following: given that defendants colluded in the conspiracy period, how much have prices been elevated above what they would have been but for the collusion? To answer this question, we need to estimate but-for prices, i.e., the prices that buyers would have paid had there been no collusion, and compare those prices

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with the actual prices. In antitrust practice, the estimated but-for prices are generally obtained from a reduced form price regression.5

In the standard framework, supply and demand are two separate functions that jointly determine price and quantity. As discussed below, in econometric terms these functions can be specified separately in two “structural equations.” Structural equations may contain both “exogenous” and “endogenous” variables. An exogenous variable is uncorrelated with the error term in the regression. This is important because the classical linear regression model assumes that the explanatory variables in a regression are uncorrelated with the error term.6 An endogenous variable is correlated with the error term in the regression. Endogeneity gives rise to “biased” estimates of the regression parameters, as discussed below. One way of handling such endogeneity problems is to combine the structural equations into a single “reduced form” equation that contains only exogenous variables. Reduced-form regressions are often used in antitrust because they can yield unbiased estimates of but-for prices.

A reduced form price regression is typically estimated using one of two approaches: the dummy variable approach or the forecasting approach.7 Both approaches regress price on demand-side and supply-side control variables. The dummy variable approach utilizes observations both from the cartel period and a non-collusive, benchmark period and includes a dummy variable for the cartel period. The forecasting approach uses only observations from the benchmark period. For the dummy variable approach, the but-for price is obtained by setting the cartel dummy to zero for the cartel period and calculating the prices predicted by the regression estimates. For the forecasting approach, estimated coefficients are used to predict but-for prices in the cartel period.


In merger cases, price regressions may be used in evaluating the effect of a merger on prices. Prices are regressed on market structure variables affected by the merger as well as demand and supply control variables. For example, in predicting the price effects of a merger between two retail chains, each local market can be characterized by number and type of chains and the number of stores each chain owns. These market structure variables would change following the merger. In addition, local markets also differ in terms of demographic variables that affect local demand and supply conditions. Store-level prices can be regressed on these market structure variables and demographic variables. With the estimated coefficients, counterfactual prices can be calculated that compare prices with and without the merger.

In both the damage analysis for price-fixing cases and merger analysis, a common criticism raised in litigation practice is that the estimated but-for prices are not reliable because one or more of the estimated coefficients in the reduced form price regression have signs inconsistent with economic theory. We discuss under what circumstances such a criticism is valid or invalid.

III. Counterintuitive Signs and But-for Price Prediction

A. Criteria for Accurate Predictions

A primary criterion used to judge the accuracy of predictions from a regression model is whether the predicted values are “biased.” A predictor is unbiased if the expected prediction error equals zero. In a linear price regression, if the estimated coefficients are unbiased then the prediction based on those coefficients is unbiased as well. Any factors that affect only the standard error of an estimate but not its expectation do no harm to the prediction according to the unbiasedness criterion.

A second criterion regarding the accuracy of a predictor is its “variance,” which measures the dispersion of predicted values. We can combine these two criteria, i.e., the bias and variance of the predicted value, by calculating the “mean squared prediction error,” which equals the variance of the predicted value plus the square of its bias. According to this criterion, we could have a poor prediction even though it is unbiased because the variance of the predicted value could be large. In a linear regression, the variance of predicted values is related to the

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variance of the estimated coefficients. \(^9\) This suggests that we need to be careful with the prediction when the counterintuitive signs are caused by large variances of the estimated coefficients. Even with unbiased price prediction, the counterintuitive signs could be signals of large variances and low accuracy of predictions.

We discuss several common interpretations of counterintuitive signs. For each interpretation, we address two questions. First, would that interpretation actually explain a counterintuitive sign, thus potentially justifying the reduced-form regression despite the counterintuitive sign, and under what circumstances would such an interpretation hold? Second, would the same interpretation lead to the conclusion that the predicted prices from the reduced-form regression are biased? In evaluating these alternative interpretations, we produce a checklist for valid interpretations of counterintuitive signs.

B. Simultaneous Equations Interpretation

The expectation of the sign of a coefficient for a given control variable is based on the relationship between price and the control variable as predicted by economic theory. Demand and cost shifters enter the demand function and the supply function, respectively. Observed prices, which are equilibrium prices, are solved by equating demand and supply. For example, suppose (1) both the demand and supply functions are linear in price and (2) there are no common variables other than price in the two functions. We can then derive the empirical, structural form of the price regression model as follows:

\[
\begin{align*}
\text{Demand function:} \quad Q_D &= \alpha_D P + X_D \beta_D + \epsilon_D, \\
\text{Supply function:} \quad Q_S &= \alpha_S P + X_S \beta_S + \epsilon_S,
\end{align*}
\]

where \(X_D\) (or \(X_S\)) is the vector of demand (or cost) shifters. Equating (1) and (2) and rearranging terms gives:

\[
P = X_D \frac{\beta_D}{\alpha_D - \alpha_S} + X_S \frac{\beta_S}{\alpha_D - \alpha_S} + \frac{\epsilon_D - \epsilon_S}{\alpha_S - \alpha_D}
\]

\(^9\) Even if the variance of an estimated coefficient is low, such that it is highly statistically significant, the value of the coefficient may not be economically significant. See, e.g., Stephen Ziliak and Deirdre McCloskey, *The Cult of Statistical Significance: How the Standard Error Costs Us Jobs, Justice, and Lives* (University of Michigan Press, 2007).
Economic theory provides predictions on the signs of coefficients in Equations (1) and (2), while the reduced-form regression model estimates the coefficients \( \frac{\beta_D}{\alpha_D - \alpha_S} \) and \( \frac{\beta_S}{\alpha_D - \alpha_S} \) in Equation (3) (the equation representing the market equilibrium).

The left and right panels of Figure 1 show, respectively, the effects of an increase in demand and supply on price. Variables that increase demand in Equation (1) will increase price in the reduced form Equation (3). Conversely, variables that increase supply in Equation (2) will decrease price in the reduced form Equation (3). Intuitively, this means that the signs of coefficients of the demand shifters \( X_D \) in the structural demand Equation (1) should have the same signs as the coefficients on \( X_D \) in the reduced form Equation (3). Conversely, the signs of coefficients of the cost shifters \( X_S \) in the structural supply Equation (2) should have the opposite signs as the coefficients on \( X_S \) in the reduced form Equation (3).

![Figure 1: Price Effects of Increases in Demand and Supply](image)

From Equation (3), the signs of the coefficients of the demand control variables \( X_D \) depend on both \( \beta_D \) and \( (\alpha_S - \alpha_D) \). In general, a higher price leads to decreased demand, so \( \alpha_D < 0 \), and a higher price leads to increased supply, so \( \alpha_S > 0 \). Together, these results imply that that \( (\alpha_S - \alpha_D) > 0 \). This implies that the coefficients on \( X_D \) in Equation (3), \( \frac{\beta_D}{\alpha_S - \alpha_D} \), should have the same sign as the coefficients on \( X_D \) in Equation (1), \( \beta_D \). Thus, in this simple case, the sign of the coefficient in the reduced-form regression model is predictable (and testable) given the economic theory. Suppose that economic theory suggests that \( \beta_D > 0 \) in Equation (1), i.e., demand increases in \( X_D \). As illustrated in the left panel of Figure 1, when increases in \( X_D \) shift the demand to the right, the equilibrium price also increases, which corresponds to \( \frac{\beta_D}{\alpha_S - \alpha_D} > 0 \).
Similarly, from Equation (3), the signs of the coefficients of the supply control variables \( X_s \) depend on both \( \beta_s \) and \( (\alpha_d - \alpha_s) \). As discussed above, \((\alpha_d - \alpha_s) < 0\). This implies that the coefficients on \( X_s \) in Equation (3), \( \frac{\beta_s}{\alpha_d - \alpha_s} \), should have the opposite sign as the coefficients on \( X_s \) in Equation (2), \( \beta_s \). Thus, in this simple case, the sign of the coefficient in the reduced-form regression model is predictable (and testable) given the economic theory. Suppose that economic theory suggests that \( \beta_s > 0 \) in Equation (2), i.e., supply increases in \( X_s \). As illustrated in the right panel of Figure 1, when increases in \( X_s \) shift the supply to the right, the equilibrium price decreases, which corresponds to \( \frac{\beta_s}{\alpha_d - \alpha_s} < 0 \).

When there is a variable that affects both demand and supply,\(^{10}\) the sign of the coefficient on that variable in the reduced-form regression may be undetermined. Let \( Z \) denote the common variable(s) and rewrite the structural demand and supply functions as

\[
\text{Demand function: } Q_D = \alpha_d P + X_D \beta_D + Z \gamma_D + \epsilon_D. \tag{4}
\]

\[
\text{Supply function: } Q_s = \alpha_s P + X_s \beta_s + Z \gamma_s + \epsilon_s. \tag{5}
\]

Then the reduced form, equilibrium price equation is

\[
P = X_D \frac{\beta_D}{\alpha_d - \alpha_s} + X_s \frac{\beta_s}{\alpha_d - \alpha_s} + Z \frac{\gamma_D - \gamma_s}{\alpha_d - \alpha_s} + \epsilon_D - \epsilon_s. \tag{6}
\]

Equation (6) shows that the sign of coefficient on the common variable \( Z \) is determined by \( \gamma_D - \gamma_s \). In some cases, economic theory cannot provide a prediction for the sign of \( \gamma_D - \gamma_s \). However, if the theory suggests that \( \gamma_D \) and \( \gamma_s \) should have opposite signs, or the theory predicts that \( \gamma_D \) and \( \gamma_s \) should have the same sign, but one coefficient should be much larger than the other, then we have a prediction of the sign of coefficient on \( Z \).

In theory, both the demand and the supply functions may not necessarily be linear or even approximately linear. For example, the demand function based on the nested logit discrete choice model is highly nonlinear.\(^{11}\) However,

\(^{10}\) For example, if the product in question is wheat, then the price of barley affects both wheat demand and wheat supply.

nonlinearity does not change our argument above and might not create any distortions in signs of coefficients if (a) the demand and supply functions are well behaved\textsuperscript{12} and (b) the relationship between the dependent variable and the independent variable is monotonic around the equilibrium. A well-behaved theoretical model, linear or nonlinear, should produce an equilibrium price function consistent with economic theory.

IV. Problems in Estimation

Counterintuitive signs in reduced-form regressions can be caused by several factors, including multicollinearity, measurement error, failing to consider indirect effects of independent variables on the dependent variable, as well as misspecification of the model.\textsuperscript{13} Thus, counterintuitive results can be caused by a variety of sources. We analyze whether a counterintuitive sign can be explained from an econometric perspective, thus potentially justifying the reduced-form regression despite the counterintuitive sign. If such an econometric explanation does not exist, counterintuitive signs may indicate fundamental problems that render the empirical results from the reduced-form regression unreliable.

A. Multicollinearity Interpretation

Particularly in price regressions, multicollinearity can cause a discrepancy between the signs of the estimated coefficients and their theoretical predictions. Suppose that two demand shifters, say GDP and unemployment, are highly correlated. The problem is that the majority of the variation in GDP and unemployment is common, and common variation is not used in estimating the coefficients on GDP or unemployment.\textsuperscript{14} Only variation specific to GDP (or unemployment), which is likely to be limited given the multicollinearity, is used in estimating the coefficient. This leads to two problems regarding the sign of coefficients. First, if only common variation captures the theoretical link between the regressors and the dependent variable, then this theoretical link is not reflected in the coefficients. For example, common variation in GDP and unemployment is supposed to capture variation in overall demand, and we expect

\textsuperscript{12} In general, well-behaved demand and supply functions are respectively downward and upward sloping. See, e.g., Hal Varian, \textit{Intermediate Microeconomics: A Modern Approach} (8th Edition), (W.W. Norton & Company, 2010).

\textsuperscript{13} For example, the reduced form price regression could be specified as linear, but the structural model could be nonlinear. In addition, the structural model may not be specified correctly relative to the underlying true model.

\textsuperscript{14} The common variation is still used in predicting the prices.
a higher price for higher overall demand. Since this common variation is not reflected in the coefficients, we might find a negative coefficient on either GDP or unemployment, or both. Second, since little variation is unique to each regressor, the coefficients would be estimated with large standard errors. Regarding the example of GDP and unemployment, the result is that the estimated coefficients can be negative due to large standard errors, while the underlying true coefficients are positive.

In addition, if estimated coefficients of the multicollinear variables are statistically significant, in which case the multicollinearity problem is not severe, the second argument regarding insufficient unique variation is unlikely to be relevant. However, the first argument, i.e., that the theoretical link is only reflected by common variation, might still apply. Thus, even if the coefficients are statistically significant (and there is no measurement error or misspecification of the model), we still might find counterintuitive signs of coefficients. Thus, if control variables are multicollinear, a reduced-form regression may still be consistent with economic theory despite the presence of counterintuitive signs.

Finally, note that this discussion is based on the unbiasedness criterion for price predictions. When the large variances in estimated coefficients of variables with multicollinearity lead to large variances in price prediction, the accuracy of the prediction is in question according to the mean squared prediction error criterion.

B. Measurement Error Interpretation

Two types of measurement error may appear when we estimate but-for prices. First, price (i.e., the dependent variable) may be measured with error. Namely, \( P = P^* + u, \text{E}(u) = 0 \), where \( P, P^*, u \) are respectively the observed prices, the true prices, and the error. We assume the error is random and independent from the true prices. For the simplicity of exposition, we assume that the prices are only affected by a single regressor \( X^* \):

\[
\begin{align*}
P &= \beta_0 + \beta_1 X^* + \epsilon_1 \\
\hat{P} &= P^* + u.
\end{align*}
\]  

The main consequence of the presence of measurement error in the dependent variable is that those errors inflate the standard errors of the
regression coefficient estimates.\textsuperscript{15} Hence, the estimated coefficient $\beta_1$ may have the opposite sign of the underlying true value due to a large standard error. Thus, if some estimated coefficients contradict economic theory, measurement error in the dependent variable is one possible explanation. However, since measurement error in the dependent variable only produces counterintuitive signs through large standard errors, it is unlikely that the problem is caused by measurement error in prices if the counterintuitive coefficient is statistically significant. In other words, if the dependent variable is measured with error, a reduced-form regression may still be consistent with economic theory despite the presence of counterintuitive signs, but likely only if the coefficients of the variables with counterintuitive signs are statistically insignificant.

The second type of measurement error is that an independent variable $X^*$ is mismeasured as $X = X^* + \nu, E(\nu) = 0$. We assume that the error is independent from the true variable $X^*$. Suppose another independent variable is $Z$, and the price regression is

$$P = \beta_0 + \beta_1 X + \beta_2 Z + \epsilon$$

where $Z$ has no measurement error. The consequence of measurement error in $X^*$ is that both $\beta_1$ and $\beta_2$ could be biased and incorrect signs may occur. Counterintuitive estimates can be due to measurement error as illustrated in this case. Unfortunately, the prediction in such a case would be biased because of the biased coefficient. In summary, if we are confident attributing the counterintuitive signs of coefficients to measurement error in an independent variable, then it is very likely the corresponding prediction is not reliable.

C. Indirect Effect Interpretation\textsuperscript{16}

A counterintuitive sign on a coefficient might be “pseudo counterintuitive” if the researcher fails to understand the difference between the total effects and the \textit{ceteris paribus} effect of that variable. By pseudo counterintuitive, we mean that the estimated sign is different from the expectation not because the estimation is in error, but because the researcher’s expectation is in error. That is, the researcher has ignored indirect effects. For example, suppose price, $P$, depends on

\textsuperscript{15} If the measurement errors have nonzero means, they will also shift the intercept even when the true slope is zero.

\textsuperscript{16} Strictly speaking, the indirect effect also can be understood as a simultaneous equations problem. For the purpose of studying reduced form price regressions, we differentiate the analysis of indirect effects from the simultaneous equation system of demand and supply functions discussed in Section III.B.
control variables $X$ and $Z$, and $Z$ affects $P$ both directly and indirectly through $X$. Economic theory predicts that the total effect of $Z$ on $P$ is positive. Then one might incorrectly argue that a negative estimated coefficient on $Z$ is counterintuitive. But this conclusion would fail to take into account the fact that the coefficient on $Z$ captures only the direct effect, which might have an opposite sign from the total effect.

However, not all indirect effects would lead to unpredictable signs of reduced-form coefficients. Consider a price regression on a grocery store competition. For simplicity, ignore other variables and potential endogeneity issues and suppose the true model is that price depends linearly on the number of grocery stores in the market and population, and the number of grocery stores in the market depends linearly on population:

$$P = \beta_0 + \beta_1 N_{\text{store}} + \beta_2 \text{Population} + \epsilon;$$

$$N_{\text{store}} = \gamma_0 + \gamma_1 \text{Population} + u.$$ \hfill (10) \hfill (11)

In addition, suppose the theory predicts that $\beta_1 < 0$, $\gamma_1 > 0$, and the total effect of population on price is positive:

$$P = \rho_0 + \rho_1 \text{Population} + \nu_1 \rho_1 > 0.$$ \hfill (12)

Then if estimation of Equation (10) gives a negative estimate of $\beta_2$, can we argue that it is reconcilable with the theory because of the indirect effect through the number of stores? The answer is no. To see this, we can substitute $N_{\text{store}}$ in Equation (11) into Equation (10) and rearrange the terms:

$$P = \beta_0 + \beta_1 \gamma_0 + (\beta_1 \gamma_1 + \beta_2) \text{Population} + \beta_1 u + \epsilon.$$ \hfill (13)

Comparing Equation (12) with Equation (13), we have $\rho_2 = \beta_1 \gamma_1 + \beta_2$. Since $\beta_1 < 0$, $\gamma_1 > 0$, and $\rho_1 > 0$, we must have $\beta_2 > 0$. Thus, an estimate of negative $\beta_2$ cannot be explained using the indirect effect argument. If there is no other good explanation as discussed in previous sections, then this suggests a modeling or data problem and cast doubts on the price prediction from the reduced-form regression.
V. Summary: A Checklist for Practitioners Problems in Estimation

To sum up our findings, we provide a checklist for antitrust practitioners. When a counterintuitive sign is found in a price regression, one can go through this list for all valid interpretations that preserve the credibility of the price prediction. If none of the interpretations on our list apply, then the counterintuitive sign is an indicator of modeling or data problems that invalidate the price predictions from the reduced-form regression.

When an estimated coefficient has a counterintuitive sign, the first thing to check is if it is statistically significant. A coefficient might be statistically insignificant because of weak dependence, not enough variation in the variable, heteroskedasticity, as well as other factors. In such cases, the estimated coefficient may be positive when the expectation is negative or vice versa due to large standard errors. However, large standard error alone will not bias the predicted price. When the variable in question has ambiguous or limited economic influence on prices, an insignificant coefficient with a counterintuitive sign generally raises less concern.

Commonly used explanations for counterintuitive signs of estimated coefficients include simultaneous equations, multicollinearity, measurement error, and indirect effects of independent variables. Table 1 summarizes the applicability and implications of these different potential interpretations.¹⁷

Lastly, whenever the check is based on statistical significance of the coefficient in question, the economist should make sure that the calculated standard errors account for data issues such as heteroskedasticity and autocorrelation. For example, if heteroskedasticity is present but ignored, then the calculated standard error is too small. The estimated regression may lead one to conclude that an estimated coefficient is statistically significant when it is not and rule out potential explanations of counterintuitive signs such as measurement error in the dependent variable.

¹⁷ Note that omitted variable bias may also cause counterintuitive signs. However, omitted variable bias generally leads to biased predictions and, therefore, cannot be used in justifying counterintuitive signs of estimated coefficients. Spurious correlation also can contribute to counterintuitive signs. Nevertheless, in practice causality between control variables is clear in most cases, and it is less likely that correlation is misinterpreted as causality. Thus, the concern of spurious correlation is less severe.
**Table 1: Applicability of Commonly Used Interpretations for Counterintuitive Signs**

<table>
<thead>
<tr>
<th>Possible Interpretations</th>
<th>Applicability and Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simultaneous equations with no common controls in demand and supply functions</td>
<td>Signs are predictable by economic theory. Counterintuitive and statistically significant coefficients suggest potential modeling or data problems.</td>
</tr>
<tr>
<td>Simultaneous equations with common controls in demand and supply functions</td>
<td>Signs are not predictable if, in theory, the common variable affects supply and demand in the same direction.</td>
</tr>
<tr>
<td>Multicollinearity</td>
<td>Estimated coefficients likely to be statistically insignificant. However, if the theoretical link between price and the multicollinear variables is only reflected by common variation, then signs are not predictable.</td>
</tr>
<tr>
<td>Measurement error in the dependent variable</td>
<td>Counterintuitive and statistically significant coefficients suggest potential modeling or data problems.</td>
</tr>
<tr>
<td>Measurement error in an independent variable</td>
<td>May be the reason for counterintuitive and statistically significant coefficients. In addition, the prediction in such a case would be biased.</td>
</tr>
<tr>
<td>Indirect Effect</td>
<td>The economist needs to understand whether the theoretical prediction relates to the total effect or the direct effect of an independent variable. Whether indirect effects would lead to unpredictable signs is case-specific and can be studied, as for example, in Section IV.C.</td>
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VI. Conclusions

Coefficients that are statistically and economically significant, and that have economically counterintuitive signs (particularly on variables that affect only demand or only supply but not both) generally indicate substantial problems that call into question the reliability of the underlying model. When the price regression model is linear, or at least well-behaved, the counterintuitive signs of coefficients are not likely explainable by the simultaneous equations argument. For price prediction to be valid, measurement errors in the dependent variable can be used to explain a counterintuitive sign only if the corresponding coefficient is statistically insignificant. Measurement errors in control variables may be the cause of a counterintuitive sign, but such measurement errors also would lead to biased price predictions. When there is no multicollinearity, the counterintuitive signs could be an indicator of model misspecification or poor quality of data, which cast doubt on predicted, but-for prices based on the reduced-form regression model. However, with multicollinearity, counterintuitive signs may arise even when the reduced-form model is valid and, thus, yields unbiased predicted but-for prices. In this case, the researcher as a check could drop one or more of the collinear control variables to ameliorate the multicollinearity issue and see if the counterintuitive sign(s) persist. If so, the reduced-form regression may well be misspecified, casting doubt on the resulting predicted but-for prices. Lastly, counterintuitive signs may result from a misunderstanding of the difference between the total effect and the *ceteris paribus* effect. The researcher should study carefully the direct and indirect effects predicted by economic theory when relying on this argument for counterintuitive signs of reduced-form coefficients.